Formal and Informal Job Search

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Abstract

I develop a model where workers decide how hard to look for a job via formal and informal search channels. The intensity of formal search determines an individual’s arrival rate of offers. The strength of investment in informal search translates into a job contact network in which job offers are transmitted. There are two equilibria, one with high formal search and one with high informal search. The former Pareto dominates the latter.

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JEL classification: A14; J64; J63; D85; E24

1. Introduction

It is well established that many workers become aware of available jobs through word-of-mouth: between 30% and 50% of jobs are filled through the use of social networks (Holzer, 1987; Galeotti and Merlino 2014, henceforth GM14). This evidence has led to several theoretical studies, which explore the importance of social networks for labor market outcomes. For example, Calvó-Armengol (2004), Calvó-Armengol and Jackson (2004), Calvó-Armengol and Zenou (2005), Chuhay (2013), Fontaine (2007) and Galeni-anos (2014).

While all these papers take the social network as given, GM14 provide a tractable model of endogenous job contact networks and show how this is key to replicate how informal search varies with labor market conditions.

In this paper I examine how workers’ incentives to search for jobs interact with their incentives to invest in their network of contacts. Indeed, different socioeconomic or ethnic groups use different search methods in their attempt to obtain information about jobs; as a result, they have very different labor market outcomes (Blau and Robins, 1990; Holzer, 1987). Yet, in GM14 workers cannot affect the arrival rate of offers by searching more intensively. More in general few papers study the interplay between choice of efforts and the formation of networks under local spillovers.1

The interaction between search effort and network investment generates multiple equilibria. While, in line with the findings of GM14, an increase in the job separation rate

1A few exceptions are Cabrales et al. (2011), Galeotti (2010), Galeotti and Goyal (2010) and Kinat-edher and Merlino (2014).
increases the network matching rate regardless of the equilibrium selected, the equilibrium with low socialization Pareto dominates the equilibrium with high socialization.

2. Model

There is a large set of risk neutral workers \( N = \{1, ..., n\} \). Initially all workers are employed and earn an exogenous wage of 1. Workers lose their job with probability \( \delta \in (0, 1) \), the job separation rate, and become job seekers. Job seekers who remain unemployed earn an unemployment benefit, which, without loss of generality, is normalized to 0.

In order to insure themselves against unemployment, workers can invest in formal and informal job search.

**Formal Job Search.** Each worker \( i \) chooses a costly search effort \( a_i \in [0, 1] \). If worker \( i \) exerts search effort \( a_i \), she accesses a direct offer with probability \( a_i \) and incurs a cost \( \alpha a_i, \alpha > 0 \).\(^2\)

Ex-ante, a representative worker anticipates that with probability \( \delta(1 - a_i) \) she will be unemployed and without a new offer. To reduce the likelihood of this event, she can engage in informal search, i.e. ask her employed friends whether they hold a needless offer.\(^3\)

**Informal Search.** An undirected network describes the diffusion of vacancies via friends. Information about jobs flows only from workers with a needless offer to job seekers with whom they have a link. A link between workers \( i \) and \( j \) is denoted by \( g_{ij} = 1 \), while \( g_{ij} = 0 \) means that \( i \) and \( j \) are not linked.

The protocol of network formation follows Cabrales et al. (2011). Each worker \( i \) chooses a costly network investment \( s_i \geq 0 \); the marginal cost of a unit of investment is constant and equal to \( c \).\(^4\)

The set of socialization efforts available to worker \( i \) is \( S_i = \mathbb{R}_+ \), and \( s_{-i} \) indicates the socialization profiles of all workers other than worker \( i \). For a profile \( s \), we assume that a link between an arbitrary pair of workers \( i \) and \( j \) forms with probability

\[
\Pr(g_{ij} = 1|s) = \begin{cases} 
\min\left\{ \frac{s_is_j}{\sum_{j \in N}s_j}, 1 \right\} & \text{if } \sum_{j \in N}s_j > 0, \\
0 & \text{otherwise.}
\end{cases}
\]

A profile \( s \) generates a multinomial random graph. When workers choose the same level of investment, say \( s \), the induced random graph is binomial.

**Utilities and Equilibrium.** Given this process of information transmission and network formation, for a pure strategy profile \((s_i, s_{-i}, a_i, a_{-i})\), let \( \Psi_i(s, a_{-i}) \) be the probability that a worker who loses her job accesses at least one offer from the network, i.e. her network matching rate.

\(^2\)Under mild regularity conditions, a more sophisticated matching function, as a Cobb-Douglas, would not affect the main results although it might affect the Pareto ranking of the equilibria.

\(^3\)Assuming that socialization takes place before job separation captures the fact that it takes time to establish connections and ensures that employed workers might have social contacts.

\(^4\)All the results we present can be derived with arbitrary cost functions \( c(s) \) that are increasing and convex in \( s \).
The expected utility to a worker $i \in N$ is

$$EU_i(s, a) = 1 - \delta(1 - a_i)[1 - \Psi_i(s, a_{-i})] - \alpha a_i - c s_i.$$  

The last two terms represents the cost of investment in the network and in direct information acquisition, respectively. The first two terms represent the probability that worker $i$ will be employed and therefore earning a wage equal to 1. This is the complement of the probability that worker $i$ is a job seeker and she neither accesses a direct offer, which happens at a rate $a_i$, nor information from the network. A pure strategy equilibrium is $(s, a)$ such that, for all $i \in N$,

$$EU_i(s_i, s_{-i}, a_i, a_{-i}) \geq EU_i(s'_i, s_{-i}, a'_i, a_{-i}), \forall (s'_i, a'_i) \in S_i \times \{0, 1\}^n.$$  

We focus on pure strategy symmetric equilibrium when $n \to \infty$, hereafter equilibrium.

3. Results

GM14 show that when $n \to \infty$, the network matching rate for worker $i$ when all other agents are playing $(s, a)$ is

$$\Psi_i(s, a_{-i}) = 1 - e^{-\frac{\alpha(1 - \delta)}{a_i}(1 - e^{-s\delta})}.$$  

Then, the following proposition characterizes interior equilibria when $\alpha < \delta$.

**Proposition 1.** Suppose $\alpha < \delta$. For every $\delta \in [0, 1]$, there exists $\alpha(\delta) > 0$ and $c(\delta, \alpha) > 0$ such that an interior equilibrium exists if and only if $\alpha \leq \alpha(\delta)$ and $c \leq c(\delta, \alpha)$. An interior equilibrium $(s^*, a^*)$ solves

$$\delta[1 - \Psi(s^*, a^*)] = \alpha \quad (1)$$
$$\delta(1 - a) \left[ \frac{\alpha(1 - \delta)}{\delta s^*} \left( 1 - e^{-s^*\delta} \right) e^{-\frac{\alpha(1 - \delta)}{s^*}(1 - e^{-s^*\delta})} \right] = c \quad (2)$$

When $c = c(\delta, \alpha)$ there exists only one interior equilibrium, otherwise there exist two interior equilibria, $(s^H, a^H)$ and $(s^L, a^H)$, where $s^H > s^L$ and $a^H > a^L$.

Equilibrium condition (1) equates the marginal cost of formal job search with its marginal returns. Marginal returns from searching decrease with the network matching rate. Hence, effort to collect information personally and network investment are strategic substitutes.

Equilibrium condition (2) equates the marginal cost of socialization with the marginal benefits represented by the increase in the network matching rate brought about by a denser network.

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5Note that if $\alpha > \delta$ there exists a unique equilibrium: workers do not search and do not invest in the network. When instead $\alpha < \delta$ there exists only one corner equilibrium (which is not stable): $s_i = 0$ and $a_i = 1$, for all $i \in N$.  

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The strategic relationship between individual search effort and network investment leads to multiple equilibria, as in Cabrales et al. (2011), although there socialization and investment are strategic complements. Equilibrium \((s^H, a^L)\) describes a labor market where workers search personally with low intensity while the social network is very dense. Low search effort can be justified only if the network matching rate is sufficiently high. Since low search effort induces a low level of job network supply (that is, few needless offers), this is attainable only if job contact networks are very dense. The reverse holds in the equilibrium \((s^L, a^H)\). Here, workers are highly engaged in collecting information personally, which is consistent with equilibrium only if the likelihood of forming connections is low, that is, the network investment is low.

Despite the multiplicity of equilibria, the analysis reveals some insights summarized in the following corollary.

**Corollary 1.** Consider the interior equilibria \((s^H, a^L)\) and \((s^L, a^H)\) described in Proposition 1. Then, (1) \((s^L, a^H)\) Pareto dominates \((s^H, a^L)\); (2) the unemployment rate under \((s^L, a^H)\) is lower than the unemployment rate under \((s^H, a^L)\); (3) the network matching rate is the same in the two equilibria and it is increasing in the job separation rate.

Some stylized facts on job contact networks surveyed in Ioannides and Loury (2004) are important to interpret this corollary. First, job contact networks display a high degree of homophily since they are formed along religious, racial, residential and occupational traits; hence, workers of different groups tend to have different contacts. Second, the usage and productivity of job contact networks varies substantially by location and demographic characteristics. Third, differences in usage do not account for all the differences in job finding rates.

Through the lenses of the model, different groups who face the same labor market condition but have somehow disconnected job contact networks might experience different labor market outcomes because they coordinate on different equilibria.

In particular, equilibrium condition (1) in Proposition 1 implies that the network matching rate is constant across the two interior equilibria and it decreases with the costs of collecting information personally. So, the model predicts that groups with lower costs of collecting information personally have higher network matching rate. This is in line with the empirical findings of Battu et al. (2011): ethnic minority groups with a higher level of country assimilation (which could be related to lower language proficiency or lower knowledge of the functioning of local labor markets) are more likely to find jobs via their social networks.

Furthermore, the equilibrium with high individual search activity and small network investment has a lower unemployment rate and Pareto dominates the equilibrium with low individual search and dense networks. Hence, relying a lot upon neighbors and personal contacts for securing employment should correlate with lower earnings. This is consistent with the finding of Elliott (1999) that less-educated urban workers in high poverty neighborhoods are more likely to use informal search than workers in low poverty neighborhoods of the same city.

Finally, an increase in the separation rate increases the network matching rate, meaning that the main correlation predicted in GM14 between labor market conditions and the proportion of vacancies filled via job contact networks is robust to endogenous formal job search intensity.
4. Conclusion

This note studies how the strategic substitutability between formal and informal job search can lead to multiple equilibria that are Pareto rankable. This is a first step towards the understanding of why different social and economic groups exposed to the same labor market conditions might rely to different extents on their social network to find jobs and might end up having different labor market outcomes.

Embedding such considerations in a full model of the labor market is a promising line of research. Indeed, studying the trade-off between formal and informal search methods in a dynamic model with endogenous wages, firms posting vacancies and search frictions in formal search would likely provide a better understanding of the labor market outcomes of disadvantaged groups and thus help in devising policies aiming at improving them.

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References

Appendix A

Proof Proposition 1 It is easy to see that an interior equilibrium \((a^*, s^*)\) solves conditions (1) and (2). I now show existence. Using condition (1) we have that \(\phi(s^*, a^*) = \alpha/\delta\), which is equivalent to
\[
a^* = \frac{\delta}{(1 - \delta)(1 - e^{-s^*\delta})} \ln \left( \frac{\alpha}{\delta} \right).
\]
Since \(a^* \in (0, 1)\) it must be the case that 
\[1 + [\delta \ln(\alpha/\delta)]/[(1-\delta)(1-e^{-s^*\delta})] > 0.\]

Let \(\alpha(\delta)\) be such that: 
\[1 + [\delta \ln(\alpha(\delta)/\delta)]/[(1-\delta)(1-e^{-s^*\delta})] = 0.\]

Note that if \(\alpha > \alpha(\delta)\) then 
\[1 + [\delta \ln(\alpha(\delta)/\delta)]/[(1-\delta)(1-e^{-s^*\delta})] \leq 0\]
for all \(s^*\), while if \(\alpha < \alpha(\delta)\) then 
\[1 + [\delta \ln(\alpha(\delta)/\delta)]/[(1-\delta)(1-e^{-s^*\delta})] > 0\]
for sufficiently high \(s^*\). So, a necessary condition for an interior equilibrium is that \(\alpha < \alpha(\delta)\).

Suppose \(\alpha < \alpha(\delta)\). Next, using the above expression for \(a^*\) and \(\phi(s^*, a^*) = \alpha/\delta\), we obtain that condition (2) holds if and only if
\[
V(\delta, c, s^*) = \frac{cs^*}{\alpha} + \ln \left( \frac{0}{\delta} \right) \left( 1 + \frac{\delta}{(1 - \delta)(1 - e^{-s^*\delta})} \ln \left( \frac{\alpha}{\delta} \right) \right) = 0.
\]
Note that \(V(\delta, c, s^*)\) is the sum of two convex function in \(s^*\) and therefore it is convex in \(s^*\). Moreover, \(\lim_{s^* \to 0^+} V(a, \delta, s^*) = \lim_{s^* \to \infty} V(\delta, c, s^*) = \infty\). Finally, note that the LHS is strictly decreasing in \(c\) and, since \(\alpha < \alpha(\delta)\), \(V(\delta, 0, s^*) > 0\). For every \(\alpha < \alpha(\delta)\), there exists a \(c(\alpha, \delta) > 0\) such that if \(c = c(\alpha, \delta)\), \(V(\delta, c, s^*) = 0\) has a unique solution in \(s^*\), while for all \(c < c(\alpha, \delta)\) the equation has two solutions, \(s^L\) and \(s^H\), with \(s^H > s^L\).

Since \(a^*\) is decreasing in \(s^*\), it follows that under \(s^H\), the equilibrium vacancy rate is \(a^H\) which is lower than the equilibrium vacancy rate \(a^L\) under \(s^L\).

Proof of Corollary 1 The equilibrium unemployment rate under equilibrium \((s^*, a^*)\) is \(u(s^*, a^*) = \delta(1 - a)\phi(s^*, a^*) = (1-a)\alpha\), where the last equality follows from condition (1). Hence, the equilibrium unemployment rate is decreasing in \(a^*\). So unemployment rate is lower under \((s^L, a^H)\) than under \((s^H, a^L)\).

Next, note that in an equilibrium \((s^*, a^*)\), \(EU(s^*, a^*) = 1 - \delta(1-a)\phi(s^*, a^*) - \alpha a^* - cs^* = 1 - a - \alpha c\), where the second equality follows from equilibrium condition (1). Hence, the expected utility in equilibrium is decreasing in \(s^*\) and therefore equilibrium \((s^*, a^*)\) Pareto dominates equilibrium \((s^H, a^L)\). Finally, from condition (1) it is immediate to see that the network matching rate is constant across the two interior equilibria and that an increase in \(\delta\) implies an increase in the network matching rate.