Discrimination, Technology and Unemployment*

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Abstract

I study the interaction between discrimination and investment using a directed search model where firms decide the capital intensity of their production technologies before being matched. Discrimination makes some workers cheap to hire. As a consequence, some firms might save on capital costs adopting labour intensive technologies. This framework allows to reconcile search models with three well-known facts regarding the labour market outcomes of minority workers: low wages, high unemployment and occupational segregation. Furthermore, the model questions the role of equal pay legislation in reducing inequality since removing this restriction, i.e., allowing firms to post type-contingent wages, eliminates the negative effects of discrimination on investment and wages.

JEL Codes: J7; J15; J16; J42.

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1 Introduction

Recent studies show that there is significant wage and unemployment inequality between African Americans and white males in the US, even controlling for observable characteristics and cognitive ability (Lang and Manove, 2011; Ritter and Taylor, 2011). Furthermore, job-cell segregation by race accounts for about half of the wage gap (Bayard and Troske, 1999), while workplace segregation occurs in a manner unrelated to observable skills (Hellerstein and Neumark, 2008). Similar differences in labour market outcomes have been found across workers defined by ethnic origin, country of birth, gender and sexual orientation.[1]

By introducing technology in a directed search model with discriminatory firms, this paper is the first to find a reasonable explanation for such employment and wage gaps, and for occupational segregation.[2] The mechanism through which wage and unemployment gaps occur is precisely occupational segregation: discriminated workers are excluded from jobs in which they are most productive, and the firms hiring them adopt less capital intensive technologies to save on investment costs.

There is consensus that discriminatory practices are still predominant and, at least to some extent, responsible for the persistence of labour outcome inequalities.[3] But if some firms discriminate, other firms can hire discriminated workers at a lower cost. For these firms, it will be less convenient to invest in labour saving technologies characterised by high capital intensity. As a result, firms that discriminate will opt for capital intensive technologies and hire non-discriminated workers, while firms that do not discriminate will be less capital intensive. Discriminated workers are employed in occupations that are less productive and whose cost-productivity ratio is higher. Hence, discriminated workers end up receiving low wages (high unemployment), since in the model this depends decreasingly (increasingly) on the cost-production ratio when there is occupational segregation.

The main message is that discriminating practices need to be studied in a framework where

[1]See, e.g., Altonji and Blank (1999), Azmat, Güell and Manning (2006), Bayard, Hellerstein, Neumark and Troske (2003), Bjerk (2007), Gardeazabal and Ugidos (2005) and Grodsky and Pager (2001).

[2]A brief discussion of the literature is provided below; see Lang and Lehmann (forthcoming) for a more detailed and exhaustive review.

[3]See Bowlus and Eckstein (2002), Flabbi (2010) and, for experimental evidence, Anderson, Fryer and Holt (2005) and Filippin and Guala (2011).

both technology and matching profiles are endogenous. In such a framework, a policy-maker aiming at reducing the negative effects of discrimination should remove equal pay requirements on posted job offers. Indeed, if firms were allowed to post type-contingent wages, workers cannot crowd out applicants from the other group by applying to wages not intended for them, and firms don't have to decide which type of workers to hire before applications are realised. As a result, the perverse effect of discrimination on investment and entry disappear and all workers get the same wage, once weighted by the individual hiring probabilities. This result might explain why unemployment and wage gaps are persistent in the US despite the introduction of equal opportunity legislation. As Fairlie and Sundstrom (1997) put it, "the persistence of the (unemployment) gap since 1960 remains an unsolved puzzle" (p. 309).[4]

The framework is the following (Section 3). All workers have the same observable and exogenous level of skills. Workers are of two groups, a and b, observable types that do not influence productivity. Firms are ex ante homogenous, and have to adopt one of the two available technologies: one is less capital intensive (hence, cheaper) and less productive, and I refer to it as the *low technology*; the *high technology* instead is more capital intensive (hence, more expensive) and more productive. After technology adoption, firms and workers engage in a wage posting game where: *(i)* firms post wages not conditioning on workers' group, say because there is an anti-discrimination regulation that makes it very costly for firms to do so; *(ii)* workers observe the distribution of offers and send their application to one of the posted wages; *(iii)* firms that received at least one applicant employ a worker deciding among candidates using a exogenous hiring rule to break ties. Discrimination takes the form that high technology firms always prefer an a applicant to a b applicant when workers of both types apply for the job.

When there is no discrimination, only the most productive technology is posted. The first and direct effect of introducing discrimination is that it makes discriminated workers less competitive, so that they are paid less. This triggers an indirect effect: since these workers are cheap, firms can attract them offering low technology jobs where workers of the dominant group do not apply.

[4] For wage, although civil rights policies aided minority workers and women in the 1960s and 1970s, the convergence has stopped in the 1980s. See Altonji and Blank (1999), Fairlie and Sundstrom (1997), and references therein.

When the productivity differential among the two technologies is small enough, firms are willing to offer these positions because, although less productive, they require less investments. For this to be an equilibrium, low technology firms should have milder hiring rules against b workers than high technology firms. This discourages a workers to apply to low technology vacancies where they face some competition of b workers. Since in the model the cost-production ratio of a technology pins down unemployment, and since this ratio is worse for the low technology, discriminated workers end up facing a high unemployment rate. Overall, while workers are paid differently because they are employed in different jobs, the equilibrium matching profile is induced by discrimination.

Introducing discrimination as a way to select applicants translates into a applicants always jumping the queue of b applicants in high technology jobs. This assumption is both weak and strong. It is weak in the sense that discrimination here is just a tie-braking rule, and hence it is not costly: firms will never forgo production to hire a applicants. It is strong in the sense that I assume that there is more discrimination in the high technology sector. There is indirect evidence to support this assumption. For example, Bjerk (2007) found that in the US minorities workers are excluded form the best occupations in the blue collar sector, while Pendakur and Woodcock (2010) found that glass ceilings for some immigrant groups in Canada are mostly driven by poor access to jobs in high wage firms. Heywood and Parent (2012) report differences in performance pay at the top of the wage distribution in the US that are consistent with discrimination and that generate white-black wage differentials among those in performance pay jobs. Furthermore, Duleep and Sanders (1992), Black, Haviland, Sanders and Taylor (2006) and Weinberger (1998) find gender and wage gaps among highly educated individuals. More in general, the glass ceilings hypothesis as been confirmed in many countries for several demographic groups.[5]

Perhaps most importantly, the discriminatory hiring rules I assume emerge endogenously in the model if there are negligible skill differentials between a and b workers with the same education and if skills are more important in high technology jobs than in low technology jobs (Section 4). If so, the optimal strategy for firms adopting the capital intensive technology would be to discriminate

[5] See, e.g., Albrecht, Björklund and Vroman (2003) and Winter-Ebmer and Zweimüller (1997) for gender, Pudney and Shields (2000) for gender and race, and Frank (2006) for sexual orientation.

and hire a workers. Low technology firms would not discriminate and employ b workers, since tight hiring rules against b workers would crowd them out more than attract a workers, reducing profits.

These results show that discrimination induces sub-optimally low levels of investment. In Section 4, I show that a social planner should remove equal opportunity legislations to promote efficient allocations.

Before deriving the main results, I will discuss the literature and present (in Section 1) a benchmark model without discrimination where firms would adopt only the most productive technology.

Related Literature

To the best of my knowledge, this is the first paper in which the interaction between discrimination and firms' investment decisions in a frictional model of the labour market is analysed. Nonetheless, these elements have been previously studied, although separately.

The closest paper is Lang, Manove and Dickens (2005), henceforth LMD. They analysed a wage posting game where homogeneous firms have a discriminatory hiring rule. As in my model, directed search induces labour market segmentation and wage gaps that greatly amplify productivity differences. Yet, since all firms must make equal profits, those who employ cheaper workers need to find them less easily, i.e., discriminated workers face lower unemployment rates.[6] Adding a pre-matching investment phase, I obtain technological dispersion and realistic unemployment gaps.

Another strand of literature uses a random search approach where firms cannot commit on wages and have strong discriminatory preferences, in the sense that they are willing to forgo production to hire non-discriminated workers. This usually results in unemployment gaps as in LMD unless strong assumptions are imposed. For example, Rosen (1997) shows that discrimination can be an equilibrium outcome when agents hold private information about their productivity and wages are set through Nash bargaining. But to obtain a realistic unemployment gap, the exogenous separation rate of discriminated workers should be higher than for non-discriminated workers. Similarly, Bowlus and Eckstein (2002) and Flabbi (2010) assume lower arrival rates for blacks to get unemployment

[6] The prediction that workers who submit applications to high wage firms should expect to be hired by those firms with low probability is common in models of directed search, i.e., Acemoglu and Shimer (2000).

differentials of the correct sign. In this paper, arrival rates and unemployment rates are endogenous, and consistent with the stylised facts. Lang and Lehmann (forthcoming) argue that introducing endogenous separation in a random search model *à la* Rosen (1997) framework should generate realistic unemployment and wage gaps, but they do not study firms' technology adoption decisions, so that in their model there is no occupational segregation.

This paper is not the first to point out that equal opportunity legislation can have perverse effects. For example, Shi (2006) showed that, when firms can condition wages on workers' type, segmentation is not an equilibrium. Kaas (2009) follows a different approach considering an economy without frictions but where nonetheless there is complementarity between the wages offered by different firms to the same group of workers. In his framework, equal opportunity induces wage gaps, but only when productivity differentials are big (or competition is strong). Here, I add to these results showing that if firms are allowed to post type-contingent contracts, the possibility of suboptimal investment and occupational segregation would disappear as well, pointing out an additional unintended consequence of an equal pay legislation: to reduce firms' investments.

Another close paper is Shi (2002), which studies technology adoption decisions in a model with skilled and unskilled workers. Here instead I concentrate on the case where all workers have the same level of skills to study the effect of a discriminatory hiring rule on investment.

In search models that endogenise technological choices, there is an inefficiently high number of low technology jobs if search is random (Acemoglu, 1998, 1999, 2001) or search costs are positive (Acemoglu and Shimer, 2000). Here instead, low investments are induced by discrimination. This difference generates very different policy implications from the ones obtained in those contexts. On the one hand, unemployment benefits would not be effective in subsidising search.[7] On the other hand, minimum wages could improve the wages paid to discriminated workers if they do not alter workers' application strategies and firms' investment. But since discriminated workers' unemployment probability needs to increase substantially due to the equal profit condition, their *ex-ante* utility would decrease.

[7] Here, every workers has a free application to send. When workers might apply for more than one job as in Albrecht, Gautier and Vroman (2006) and Galenianos and Kircher (2009), the analysis would get more complicated, but the main results presented in this paper would not change since a workers will tend to apply more to high paying jobs.

2 The Benchmark Model

The benchmark model is a labour market with a mass of N of risk neutral homogeneous workers, all endowed with the same level of observable skill, and a mass M of risk neutral firms that can post different types of vacancies and do not use discriminatory hiring rules. M is determined by free entry. This is a standard wage posting game model with a pre-matching investment phase where firms are potentially ex-post heterogeneous due to the choice of technology adoption. The framework developed in this section is similar to Shi (2002), but with homogeneous workers.

There are two technologies: a low technology involving an easy task which yields a quantity y of output if the vacancy is filled, and a complex technology, which yields θy units of output, $\theta > 1$.

The two technologies have different capital intensities: a high technology job costs K^H to set up, while a low technology job costs $K^L < K^H$. I will assume $\theta > K^H/K^L > 1$ and $K^L < y$. Namely, the productivity in the high technology sector is assumed to be sufficient to cover the higher investment cost of the capital intensive high technology firm, while posting a labour intensive low technology vacancy is cheaper but more costly relatively to the productivity of workers.

I will denote by H the endogenous fraction of firms that use a high technology. Abusing notation, a subscript $J \in \{H, L\}$ will also indicate high technology firms and low technology firms respectively.

Each firm has only one job opening and each worker applies to one job.

The timing is as follows. First, firms decide whether to enter the market and which technology to adopt. Second, all firms simultaneously post a wage schedule. Third, after observing posted wages, all workers choose which job to apply to. Since they do not coordinate on which firm to apply to, they apply randomly to one of the firms posting a certain wage. Finally, each firm of type J selects a worker at random from the pool of applicants (if any), production takes place according to $Y(J)$, and the firm pays the announced wage to the worker. Unmatched firms and workers are not productive, and hence get a payoff of zero.

I will focus on the symmetric, mixed strategy equilibrium where ex-ante identical agents use the same strategy.[8] The strategy of a firm j consists of posting a vacancy of type J, $J = H, L$, and

[8]This restrictions captures the difficulty that agents have to coordinate themselves in a large labour market. See

a committable wage contract w^J, where $w^J \in \mathbb{R}_+$. Denote then the measure of firms posting each possible wage w^J by $f(w^J)$, which summarises the strategies of all firms.

Workers observe the proportion of firms posting each contract w^J, described by f, and define their strategies as the probability $p(w^J|f)$ to apply to each one of these contracts.

Since there is a continuum of firms and workers, the fraction of firms posting a certain wage might approach zero. Given f, p and w^J, I use the queue length $q(w^J|f,p)$ at a firm of type J posting the wage w^J as a more convenient way of describing the aggregate behaviour of workers. This is defined as the expected measure of workers applying to firms posting this menu (which depends on the strategies of workers and their mass), divided by the measure of firms posting the same menu. Then, for $J = L, H$ and w^J such that $f(w^J) > 0$,

$$q(w^J|f,p) = \frac{p(w^J|f)N}{f(w^J)M}.$$

Given f and p, let $\mu(w^J|f,p)$ be the probability that a worker gets a job in a J firm posting w^J. Namely,[9]

$$\mu(w^J|f,p) = \frac{1 - e^{-q(w^J|f,p)}}{q(w^J|f,p)}. \tag{1}$$

The term $1 - e^{-q(w^J|f,p)}$ is the probability that at least one applicant shows up at such a firm. Dividing this by the expected measure of applicants to that type of firm, $q(w^J|f,p)$, one gets the probability that this particular worker is chosen among the pool of applicants of the same type. When

Burdett, Shi and Wright (2001) for a more detailed discussion.

[9]There are many ways to derive expression (1). One is the following. Since workers' strategies are independent, the realizations of the application process are independent across workers. Then, given f and p, the actual number of received applications in a firm j posting w^J is a Poisson process. Hence, a firm receives $z \in \mathbb{N}$ applicants with probability

$$Prob(z|w^J,f,p) = \begin{cases} \frac{1}{z}[q(w^J|f)]^z e^{-q(w^J|f)} & \text{if } q(w^J|f) > 0, \\ 0 & \text{otherwise.} \end{cases}$$

When there are other z applicants, a worker is chosen with probability $1/z+1$. Summing across z gives the hiring probability, conditional on applying for the job, of

$$\begin{aligned} \mu(w^J|f) &= \sum_{z=0}^{\infty} \frac{1}{(z+1)!}[q(w^J|f)]^z e^{-q(w^J|f)} \\ &= \frac{1-e^{-q(w^J)}}{q(w^J)}. \end{aligned}$$

no confusion may arise, I will write $q(w^J)$ instead of $q(w^J|f,p)$ and $\mu(w^J)$ instead of $\mu(w^J|f,p)$.

The probability to obtain a job is strictly decreasing in the queue of applicants: the larger the number of applicants, the lower the probability to get the job, since this is uniformly distributed among them.

For $J = H, L$, let $v(w^J|f,p)$ be the market utility associated with applying to the contract w^J; namely, $v(w^J|f,p) = \mu(w^J)w^J$. I will often write $v(w^J)$ instead of $v(w^J|f,p)$.

For workers to apply for a job, the expected value of the offer has to be at least as large as the expected income of workers, denoted by V, the worker market utility. Observe that $V \geq 0$, since the market utility of unmatched agents is equal to zero. The expected value of the offer is the wage paid times the probability to get the job once one has applied to. Thus, for $J = H, L$,

$$V \leq v(w^J) = \mu(w^J)w^J.$$

However, if $0 \leq V < \mu(w^J)w^J$, all workers will apply to that position with probability 1, yielding $\mu(w^J) = 0$, a contradiction. Thus, given f and p, for $J = H, L$, $q(w^J)$ can be written as a functional associating to a contract w^J posted by $Mf(w^J)$ firms a certain queue length, which is strictly positive only if $V = v(w^J)$. Namely, $q(w^J) > 0$ if $v(w^J) = V$ and $q(w^J) = 0$ if $v(w^J) < V$.
Using this result and (1), I can derive the expression for the wage that a J firm sets if $q(w^J) > 0$ as $w^J = Vq(w^J)/\left[1 - e^{-q(w^J)}\right]$.

Finally, the expected gross profits of a firm j posting w^J before paying the cost K^J of opening a vacancy can be written as $\pi_J(w^J|f,p) = \left[1 - e^{-q(w^J)}\right](Y(J) - w^J)$, given f and p.[10] The first term of such expression is the probability that at least one worker shows up. The second is the net benefit of the job, that is, productivity minus the wage paid. I can substitute wages into the expression for profits, obtaining:

$$\pi(w^J) = \left[1 - e^{-q(w^J)}\right] Y(J) - Vq(w^J), \tag{2}$$

[10] I will assume throughout the model that firms are a single profit-maximizing entity. A different approach is taken by Méon and Szafarz (2011), who propose a principal-agent model of labour market discrimination where managers are discriminatory and do not maximise shareholders' profits.

meaning that the price firms need to pay for an increase in their queue lengths is the reservation rule of the applicants given by their market utility V. Again, when no confusion may arise, I will write $\pi(w^J)$ instead of $\pi_J(w^J|f,p)$.

Given the timing of the wage posting game, a firm decides which contract to post considering the queue of applicants that the contract is going to attract, i.e. it solves the game using backward induction. Hence, a firm can offer a higher wage to increase its matching probability: a marginal wage increase attracts a marginal increase in the expected number of applicants. All applicants do not necessarily apply to the higher wage because they take into account the probability to obtain that job, which is decreasing with the queue lengths.

2.1 Definition of Equilibrium

The economy can be fully described by an allocation as defined below.

Definition 1 *Given N, an allocation is given by a tuple (M, f, q, V, π) where*

1. *M is the measure of firms*;
2. *f is a function describing the measure of firms posting each contract w^J for $J = H, L$;*
3. *for $J = H, L$, $q(w^J)$ is a function describing the queue length associated to each posted vacancy induced by the probability to apply to each contract as described by p;*
4. *V is a function describing workers' market utility;*
5. *π is the level of net profits of firms.*

The functional $q(w^J)$ determines the queues not only for wages actually posted in the market, but also for all possible wages, determining the out-of-equilibrium-path actions which help to pin down equilibrium behaviour. When there will be few contracts posted in equilibrium, the distribution function f will be degenerate and it will be enough to consider the proportion of firms posting each wage.

Definition 2 *A competitive search equilibrium is an allocation* $(M^*, f^*, \pi^*, q^*, V^*)$ *such that:*

1. *Profit Maximization: for all* $w^J \in \mathbb{R}_+$ *and* $J = H, L$, $\pi(w^J) \leq K^J$ *with equality if* $f^*(w^J) > 0$.

2. *Optimal Applications: workers' market utility is given by* $V^* = \sup_{w^J | f^*(w^J) > 0, J=H,L} \mu(w^J | f, p) w^J$.

3. *Slackness condition: for all* (w^J), $J = H, L$, $\mu^*(w^J) w^J \geq V^*$ *with equality if* $q^*(w^J) > 0$. *The probability* $\mu^*(w^J)$ *is computed using (1) with the equilibrium queues.*

4. *No Drop Out:*
$$\int q^*(w^H) df^*(w^H) + \int q^*(w^L) df^*(w^L) = N/M^*. \tag{3}$$

The first condition ensures that firms are posting wages and selection criteria optimally, given the application strategies of workers. Furthermore, because of free entry, every job offer posted in equilibrium gives zero net profits to firms. Note that gross profits in the two sectors will differ because of the different cost of opening a vacancy. Conditions 2 and 3 ensure that the application strategies of workers are chosen optimally given posted wages and the market utility of each type. The last condition is a consistency requirement ensuring that each worker applies to one job opening.

The following Theorem describes the equilibrium of such a market. For $x \in \mathbb{R}$, let $\beta(x) = B^{-1}(x) \equiv 1 - (1 + x)e^{-x}$. Then,

Theorem 1 *There is a unique symmetric equilibrium of the labour market. In equilibrium, no L vacancies are posted* ($H^{nd} = 1$) *and all H firms post the same wage,* w^{nd}. *Queues, measure of firms and market utility are given by:*

$$q(w^{nd}) = N/M, \tag{4}$$
$$V^{nd} = \theta y e^{-N/M}, \tag{5}$$
$$M^{nd} = \frac{N}{B(K^H/\theta y)}. \tag{6}$$

Hence, the introduction of a pre-matching investment phase does not change the results of a classical wage posting game. There is a unique equilibrium where only the most efficient technology is posted and all firms post the same wage. Low technology vacancies are not offered because workers are more productive in high technology firms. The cost of opening a low technology vacancy is lower, but also the productivity of a match. As a result, in order to attract applicants, low technology firms should offer a wage that is too expensive for them. This result follows from the assumptions that $\theta > K^H/K^L > 1$ and $K^L < y$, which simply state that the low technology is not so cheap to be preferred to the high technology.

3 The Model with Weak Discrimination

I will now introduce discriminatory hiring rules, and show that this has a big effect not only on the wages paid to workers, but also on equilibrium matching profiles.

Assumption 1 ***(Workers' Types)*** *There are two types of workers, a and b; the proportion of workers of a is $\alpha \in (0,1)$.*

Assumption 2 ***(Equal opportunity)*** *Wages cannot depend on workers' types.*

Assumption 3 ***(Weak Discrimination)*** *Type a workers are preferred to b workers in the high technology sector.*

Assumption 1 simply says that workers differ only on the way they are labelled. Assumption 2 asserts that firms cannot condition their wages on workers' types. This means that a firm cannot post different wages to equally productive workers employed in the same position just because of their label. This could be because of some equal opportunity legislation. Finally, Assumption 3 says that firms use a hiring rule that is systematically biased towards one of the two types in the high technology sector. Since the wage menu cannot be discriminatory, the only way firms can discriminate is by using a particular hiring rule. I do not restrict the hiring rule in the low technology sector: I will show below that, for the existence of an equilibrium where discriminated workers apply to the low technology sector, there need to be a discrimination differential between the two sectors. The interpretation would be that, while there is a prejudice that minority workers cannot/are not suited to perform complex tasks, they are considered adequate to perform easy tasks. I will discuss below when it would be optimal for firms to use such hiring rules.

For $J = L, H$, the queues of workers belonging to the two types now are

$$q_a(w^J|f,p) = \frac{\alpha N p_a(w^J|f)}{f(w^J)M} \qquad\qquad q_b(w^J|f,p) = \frac{(1-\alpha)N p_b(w^J|f)}{f(w^J)M},$$

where $p_i(w^J|f)$ is the mixed strategy describing with which probability a worker of type i, for $i = a, b$, will apply to a firm offering a wage w^J for $J = H, L$.

I will capture Assumption 3 assuming different hiring rules, x^H and x^L, in the two sectors. Hence, for $J = H, L$, I can then write the probability that an a worker obtains a job as

$$\mu_a(w^J|f,p) = \frac{1 - e^{-q_a(w^J|f,p)}}{q_a(w^J|f,p)} \left[x^J + (1 - x^J)e^{-q_b(w^J|f,p)}\right]. \tag{7}$$

Hence, $x^H = 1$, so that a workers only care of applicants of their type when applying to a job in this sector. On the other hand, for the moment I consider $x^L \in [0,1]$, meaning that a applicants face at least to some extent the competition of b applicants. A symmetric argument holds for b applicants:

$$\mu_b(w^J|f,p) = \frac{1 - e^{-q_b(w^J|f,p)}}{q_b(w^J|f,p)} \left[1 - x^J + x^J e^{-q_a(w^J|f,p)}\right]. \tag{8}$$

As usual, the probability to obtain the job is strictly decreasing in the queue of b applicants. But also the expected queue of a applicants matters through the term $1 - x^J + x^J e^{-q_a(w^J|f,p)}$, which indicates how much it is important that no a worker shows up for a vacancy, an event that occurs with probability $e^{-q_a(w^J|f,p)}$.

For $i = a, b$ and $J = H, L$, let $v_i(w^J|f,p)$ be the market utility for a worker of type i applying to a contract w^J which can be computed as $v_i(w^J|f,p) = \mu_i(w^J|f,p)w^J$.

When no confusion may arise, I will write $q_i(w^J)$ instead of $q_i(w^J|f,p)$, $\mu_i(w^J)$ instead of $\mu_i(w^J|f,p)$ and $v_i(w^J)$ instead of $v_i(w^J|f,p)$, for $i = a, b$ and $J = H, L$.

As in the benchmark model, for workers of type i to apply for a position, the expected value of the offer has to be at least as big as workers' market utility, V_i, for $i = a, b$. Thus, for $J = H, L$ and $i = a, b$, if $q_i(w^J) > 0$, wages can be written as

$$w^J = \frac{V_i}{\mu_i(w^J)} \tag{9}$$

Hence, when a and b workers apply to the same firm, they apply to a job paying the same wage, as required by Assumption 2.

The expected profits of a firm adopting technology $J \in H, L$ posting a contract w^J are:

$$\begin{aligned} \pi^J(w^J) &= \mu_a(w^J)q_a(w^J)(Y(J) - w^J) + \mu_b(w^J)q_b(w^J)(Y(J) - w^J) \\ &= (1 - e^{-q_a(w^J) - q_b(w^J)})Y(J) - V_a q_a(w^J) - V_b q_b(w^J). \end{aligned} \tag{10}$$

The first term in the first line of the expression for profits is the probability that at least an a applicant shows up and gets a job times the net output going to the firm. The same is true for b workers. Since each worker is equally productive, expected production is given by the probability of filling a vacancy with any type of worker by the productivity of the match, independently of the hiring rule adopted.

3.1 Definition of Equilibrium

The definition of allocation and equilibrium in this economy is as in the benchmark model, once taken care of the fact that now there are two types of workers.

Definition 3 *Let N, $x^H = 1$, $x^L \in [0,1]$ and α be given. An allocation is given by a tuple $(M, f, q_a.q_b, V_a, V_b, \pi^L, \pi^H)$ where*

1. *M is the measure of firms posting vacancies;*

2. *f is a function describing the measure of firms posting each contract w^J for $J = H, L$;*

3. q_i *is a function describing the queue length associated to each posted vacancy, for* $i = a, b$;

4. V_i *is a function describing the market utility of workers of type* i*, for* $i = a, b$;

5. π^J *is the function describing the profits obtained by a firm posting a contract* w^J*for each* $J = H, L$.

Hence,

Definition 4 *A competitive search equilibrium is an allocation* $(M^*, f^*, q_a^*, q_b^*, V_a^*, V_b^*, \pi^*)$ *such that:*

1. *Profit Maximization and Free Entry: for all* $w^J \in \mathbb{R}_+$ *and* $J = H, L$, $\pi^J(w^J) \leq K^J$*, with equality if* $f^*(w^J) > 0$.

2. *Optimal Applications: for* $i = a, b$, $V_i^* = \sup_{w^J | f^*(w^J) > 0, J = H, L} \mu_i(w^J | f) w^j$.

3. *Slackness condition: for all* w^J, $i = a, b$ *and* $J = H, L$, $q_i^*(w^J)\left[\mu_i^*(w^J) w^j - V_i^*\right] = 0$*, where* $\mu_i^*(w^J)$ *is computed by (7) and (8) introducing the equilibrium queues.*

4. *No Drop Out:*

$$\int q_a^*(w^H) df^*(w^H) + \int q_a^*(w^L) df^*(w^L) = \alpha N / M^*,$$

$$\int q_b^*(w^H) df^*(w^H) + \int q_b^*(w^L) df^*(w^L) = (1 - \alpha) N / M^*.$$

These definitions have the same meaning of their parallels in the benchmark model.

3.2 Equilibrium Characterization

To characterise the equilibrium of the model, I will first show that, when both a and b workers apply to high technology firms, this sector of the market will segment in high wage firms attracting a workers and low wage firms attracting b workers. In other words, the discriminatory hiring rule distorts the employment opportunities of b workers making them willing to accept lower wages not to face the competition of a workers in the hiring process.

In the low technology sector, a firm will never attract a applicants. Indeed, the decisions of a applicants are unaffected by the presence of b workers: the former will always be preferred to the latter in the queues.

Discrimination instead affects the allocation of the workers that are discriminated: they might be offered positions in the low technology sector because discrimination lowers their market wage enough so that low technology firms can attract them.

Whether this happens or not in equilibrium depends on the productivity gap between the two sectors. Intuitively, for the low technology to be profitable, it must be that its productivity is not too small with respect to the other technology. Furthermore, there has to be a "discrimination gap", that is, a workers should face some competition with b workers when applying for low technology positions.

In both equilibria, there are two kind of firms: firms attracting only a workers and firms attracting only b workers. I will denote by w_a^J and w_b^J the wage posted respectively in sector J, for $J = H, L$.

The following proposition shows that a and b workers never apply to the same contract in the H sector.

Proposition 1 *When a and b workers apply to H firms, only two equilibrium wages are posted, each attracting one type, i.e., for all w^H such that $f(w^H) > 0$, either $q_a(w^H) = 0$ or $q_b(w^H) = 0$, and*

$$w_a^H = \frac{\theta y e^{-q_a(w^H)} q_a(w^H)}{1 - e^{-q_a(w^H)}}, \tag{11}$$

$$w_b^H = V_a, \tag{12}$$

$$V_a = \theta y e^{-q_a(w_a^H)}, \tag{13}$$

$$V_b = \frac{1 - e^{-q_b(w_b^H)}}{q_b(w_b^H)} V_a, \tag{14}$$

The proof of Proposition 1 is an adapted version of the proof of Propositions 2 and 3 in LMD. Firms post either a high wage w_a^J to which only a workers will apply or a low wage w_b^J to which only b workers will apply.

Furthermore, the following remarks further characterise equilibrium behaviour.

Remark 1 *In equilibrium,* $q_a(w_a^H) > 0$.

It will be always profitable to offer a vacancy in the high technology sector to a workers. On the other hand, b workers might apply or not to high technology positions depending on the wages offered in the low technology sector.

Remark 2 *In equilibrium,* $q_a(w_a^L) = 0$.

Since a workers behave exactly as in the benchmark model, their market utility is so high that they cannot be in profitably employed in low technology vacancies.

These results imply that the labour market segments, and that b workers might apply either to the high technology sector or to the low technology sector. The following theorem describes when each situation can emerge. Denote $C\left(K^H/\theta y\right) = \log(e^{B\left(K^H/\theta y\right)} - 1) - \log B\left(K^H/\theta y\right)$. Furthermore, I will denote the proportion of firms attracting a workers in the H sector by $H_a \equiv f(w_a^H)$. Then:

Theorem 2 *In the model with Weak Discrimination, there is a unique equilibrium. All workers of type a apply to H vacancies. Their wages and market utility are as in (11) and (13), while queues are given by:*

$$q_a^* = B\left(\frac{K^H}{\theta y}\right). \tag{15}$$

Furthermore, if $\theta \leq \bar{\theta}(y, K^H, K^L)$ and $x^L \leq \bar{x}(y, \theta, K^H, K^L) < 1$, all workers of type b apply to L vacancies. Queues, the proportion of firms posting the high wage and the measure of firms are given by:

$$q_b^* = B\left(\frac{K^L}{y}\right), \tag{16}$$

$$H_a^* = \frac{\alpha B(K^L/y)}{(1-\alpha)B(K^H/\theta y) + \alpha B(K^L/y)}, \tag{17}$$

$$M^* = N\frac{(1-\alpha)B(K^H/\theta y) + \alpha B(K^L/y)}{B(K^H/\theta y)B(K^L/y)}. \tag{18}$$

Otherwise, b workers apply to H vacancies. Wages and market utilities are as in (12) and (14), where queues, the proportion of firms posting the high wage and the measure of firms are given by:

$$
\begin{aligned}
q_b^{**} &= C\left(\frac{K^H}{\theta y}\right), && (19)\\
H_a^{**} &= \frac{\alpha C\left(K^H/\theta y\right)}{(1-\alpha)B\left(K^H/\theta y\right)+\alpha C\left(K^H/\theta y\right)}, && (20)\\
M^{**} &= N\frac{(1-\alpha)B\left(K^H/\theta y\right)+\alpha C\left(K^H/\theta y\right)}{B\left(K^H/\theta y\right)C\left(K^H/\theta y\right)}. && (21)
\end{aligned}
$$

Let us first concentrate on the case when all b workers apply to the low technology sector, even if they are more productive in the high technology sector. Since $K^L < K^H$, this equilibrium predicts that discriminated workers are employed in less capital intensive firms with respect to non discriminated workers. The wage gap penalises discriminated workers, and, most importantly, they face a higher unemployment rate, given that $B\left(K^L/y\right) > B\left(K^H/\theta y\right)$.

Only a workers apply to the high technology sector, and only b workers apply to the low technology sector. Because of free entry, the high technology sector is affected through H_a^* and M^*. The proportion of firms posting vacancies in the high technology sector is increasing in the proportion of a workers, α, which are the ones that are going to apply to these vacancies. The denominator of (17) indicates that the proportion depends also on relative costs of the two technologies, weighted by the proportion of applicants in the other sector. So, when the cost of opening a high technology vacancy increases, $B(K^H/\theta y)$ will increase, decreasing H_a^*. An increase in K^L would have the opposite effect, and the relative size of these effects depends on α, which is the proportion of a workers. The total measure of firms is obviously increasing in the measure of workers, N.

Figure 1 depicts such an equilibrium. Firms posting different types of vacancies now post contracts such that the isoprofit line and the workers' market utility are tangent. But, due to the difference in the level of investment required to adopt the two technologies, they are on different isoprofit lines. As a result, unemployment and wage gap go in the same direction.

Conditions $\theta \leq \bar{\theta}(y, K^H, K^L)$ and $x^L \leq \bar{x}(y, \theta, K^H, K^L)$ ensure that the equilibrium exists by making sure that firms and workers have no profitable deviation. Indeed, b workers should not find

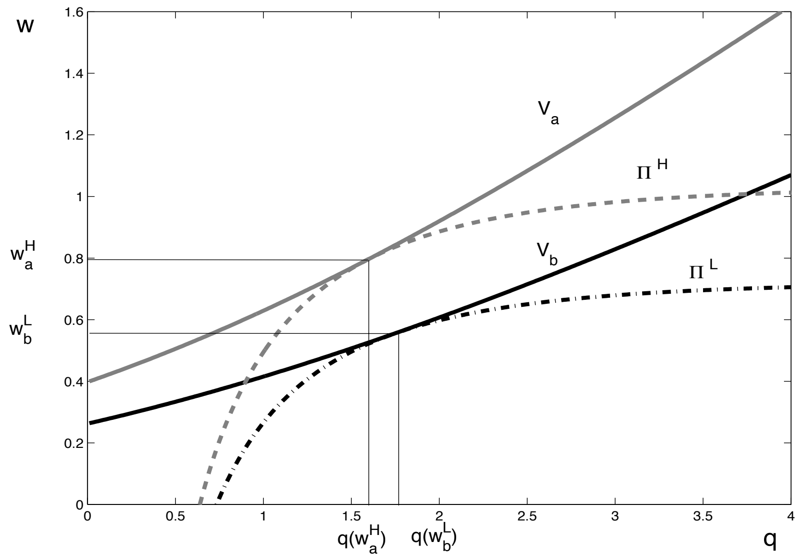

Figure 1: A graphic representation of the equilibrium when a workers apply to high technology positions while b workers apply to low technology positions.

it profitable to try to get a job in the high technology sector, which does not happen if $V_b^* > e^{-q_a^*} w_a^*$.

Then, firms should not find it profitable to offer a job to b workers in the high sector. This is true whenever a firm offering the same expected wage in the H sector would incur negative profits. Hence, we need to impose $ye^{-B(K_L/y)} \geq V_b^{**}$, if not firms could offer V_a^* to b workers and still get positive profits in the H sector. This condition, which subsumes the first one, translates into

$$e^{-B\left(\frac{K^L}{y}\right)} \geq \frac{1 - e^{-C\left(\frac{K^H}{\theta y}\right)}}{C\left(\frac{K^H}{\theta y}\right)} \theta e^{-B\left(\frac{K^H}{\theta y}\right)}. \tag{22}$$

Since the left hand side of this expression is continuous and strictly increasing in θ, there exists a $\bar{\theta}(y, K^H, K^L)$ such that (22) is satisfied for all $\theta \leq \bar{\theta}(y, K^H, K^L)$.

Finally, in equilibrium a workers should not be willing to apply for a position in the low technology sector. This is true whenever $V_a \geq [x^L + (1-x^L)e^{-B\left(K^L/y\right)}]w_b^*$. This implies

$$\theta e^{-B\left(\frac{K^H}{\theta y}\right)} \geq \frac{B\left(\frac{K^L}{y}\right)e^{-B\left(\frac{K^L}{y}\right)}}{1-e^{-B\left(\frac{K^L}{y}\right)}}[x^L + (1-x^L)e^{-B\left(\frac{K^L}{y}\right)}]. \tag{23}$$

When $x^L = 1$, (23) and (22) are incompatible. When $x^L = 0$, (23) is always true if (22) is. Since $x^L + (1-x^L)e^{-B\left(K^L/y\right)}$ is continuous and increasing in x^L, there exist a unique $\bar{x}(y, \theta, K^H, K^L)$ such that for all $x^L \leq \bar{x}(y, \theta, K^H, K^L)$, (23) is true. Hence, x^L must be strictly lower than 1, i.e., discrimination must be milder in the low technology sector. I will show later that, if x^H and x^L were endogenous, under mild conditions such hiring rules are optimal for firms.

If one of these two conditions is not satisfied, both a and b applicants look for a job in the high technology sector. In such case, the model has the same features as the labour market with discrimination and homogenous firms analysed by LMD.

In particular, $C\left(K^H/\theta y\right) < B\left(K^H/\theta y\right)$, which translates into the counterfactual results that discriminated workers have lower unemployment rates.

The conditions for this equilibrium to exist result from ruling out that firms can profitably offer a contract in the low technology sector that attracts b workers. This translates in the fact that a firm offering a market wage as the one paid in the high technology sector should obtain negative profits. Hence, these conditions are the counterpart of the deviations analysed above: either the low technology sector is not productive, or offering a wage in that sector attracts enough a workers to crowd out b applicants.

Figure 2 depicts this equilibrium. Since all firms are posting the same type of vacancy, they are on the same isoprofit line. As a result, while a workers receive a non distorted contract (they are not influenced by the presence of b workers), b workers' wage is such that a workers are just indifferent between applying or not to that wage, that is V_a. As a result, $q_a(w_a^H) > q_b(w_b^H)$, that is, a workers face a higher unemployment rate, to compensate firms of the higher wage they receive.

To conclude, the interaction between firms' investment decisions, discrimination and workers' directed search generates a novel and interesting equilibrium where both unemployment and wage

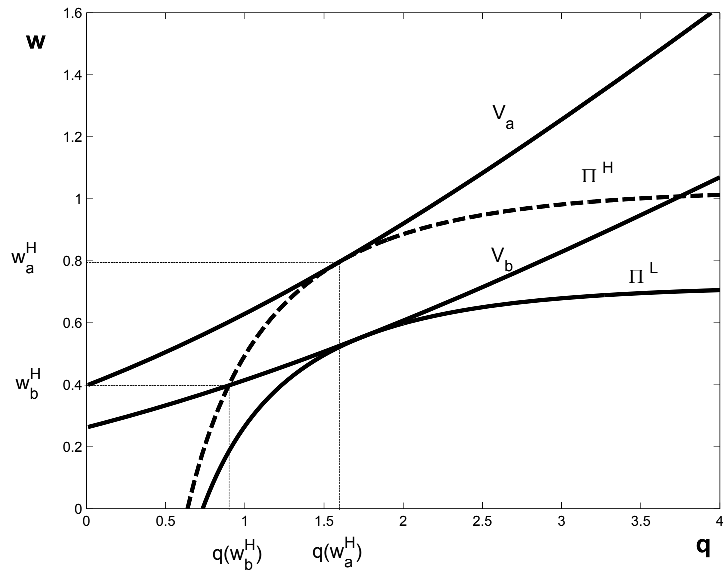

Figure 2: A graphic representation of the equilibrium when only high technology positions are offered.

gaps are realistic. The mechanism is that discrimination makes some workers easy to attract and this makes profitable a technology that is not the most productive.

This implies correlations that have some interesting implications in the way one can detect discrimination. First of all, if a researcher takes the matching profile as random, he would filter out firms' characteristics to determine whether different types of workers have different labour market outcomes. Suppose such a researcher uses data that come from the equilibrium where low technology vacancies are posted. Then, he would attribute the differences in the labour market outcomes of a and b workers as a firm level effect.

Second, conditioning on educational level, discriminated workers are employed in sectors that are less capital intensive. While discriminated workers are usually employed in sectors that are more capital intensive because these are more traditional sectors, by conditioning on observable workers' characteristics, the opposite should hold. In other words, when the matching profile is endogenous, the fact that some workers are systematically employed in less productive firms can be symptomatic of discrimination. Hence, my model provides a novel instrument to detect whether wage gaps associated with occupational segregation are due to discriminatory practices.

4 Discussion

Hiring rules. The equilibrium where low technology jobs are offered is especially interesting because it allows to rationalise the fact that discriminated workers have both lower wages and higher unemployment. This suggests that discrimination acts on the matching profile when we allow firms to exert some pre-match investment. I will now show that, when the low technology is productive enough, small productivity differentials in the H sectors would be enough to generate hiring rules as the ones that sustain the equilibrium described above.

I have assumed above that all workers are equally productive in the H sector. If instead b workers are even just slightly less productive in the H sector, hiring rules necessary to obtain the two equilibria described above are the optimal ones as the following Corollary shows.

Corollary 1 *Suppose x_J, $J = L, H$ are part of the posted contracts and b workers are $\epsilon > 0$ less productive than a workers. Then, if $\theta \leq \hat{\theta}(y, K^H, K^L, \epsilon) < \bar{\theta}(y, K^H, K^L)$, firms would adopt $x^H = 1$ and $x^L \leq \hat{x}(y, \theta, K^H, K^L, \epsilon)$ and b workers would be working in the L sector. Otherwise, firms would adopt $x^H = 1$ and $x^L \in [0, 1]$ and b workers would be working in the H sector.*

Hence, which equilibrium emerges depends only on the productivity differential across sectors. The result that $x^H = 1$ follows directly from the productivity differential in the H sector. The result on x^L follows from the fact that posting a vacancy to attract only b workers in the H sector would not be profitable. Indeed, $\theta \leq \hat{\theta}(y, K^H, K^L, \epsilon)$ ensures that the market utility of b workers is so high that a firm would not attract enough of them to make non-negative profits.

Small productivity gaps across workers can be rationalised following the logic of Lang and Manove (2011), which also provide evidence of such gaps. If firms observe the productivity of b workers less accurately than that of a workers, they would put more weight on education and less on observed productivity for b workers. Hence, they would get more education. As a result, given the level of education, b workers should be of lower ability. This has an impact in the high technology sector where workers have to perform complex tasks. Since in the low technology sector workers are required to perform easy tasks, ability does not matter there. Overall, directed search would greatly amplify these small productivity differences because of their impact on the hiring selection rules of firms.

Production. Since the two equilibria exist for different sets of parameters, one cannot directly compare expected production in the two. So, I do the following thought experiment: would production be higher or lower if firms could only adopt the most advanced technology? While employed b workers would produce more, the measure of active firms and queues would change, so that the overall effect is not obvious. The following proposition answers this question.

Proposition 2 *Suppose $\theta \leq \bar{\theta}(y, K^H, K^L)$. Then, production is higher if $x^L > \bar{x}(y, \theta, K^H, K^L)$.*

While workers are employed in less productive jobs, unemployment is higher, which means that for firms it is easier to find workers.

Equal Opportunity. It turns out that the effects of discrimination on workers market utilities and technology adoption depend on the assumption of Equal Opportunity. When firms are free to discriminate wages, firms would be able to target wages to different workers. By posting two wages instead of only one, the trade-off between attracting a workers and deterring applications from b workers disappears because firms can compensate workers with longer queues paying higher wages. All workers receive the same market utilities and it would be too expensive to attract workers to the low technology sector. Hence, Lemma 2 would go through for both workers' types. The result is summarised in the following proposition.

Proposition 3 *In an economy with Weak Discrimination but no Equal Opportunity, only H jobs are posted and both workers obtain the same market utility. Queues and the measure of firms are given by:*

$$\widetilde{q}_a = \alpha B\left(\frac{K^H}{\theta y}\right), \tag{24}$$

$$\widetilde{q}_b = (1-\alpha) B\left(\frac{K^H}{\theta y}\right), \tag{25}$$

$$\widetilde{M} = \frac{N}{B(K^H/\theta y)}. \tag{26}$$

As noted by Shi (2006), when firms are free to discriminate wages and workers have slightly different productivities, the optimal selection rule would be again discriminatory. Furthermore, wages might actually be higher for discriminated workers; in that case, firms are compensated by a high arrival rate of these workers and market utilities are such that discriminated workers earn less than the others.

This message is particularly important since models without discrimination that generate misallocations yields very different policy implications. For example, Acemoglu (2001) proposes minimum wages and unemployment benefits to reduce misallocation inefficiencies that are present in his model even when the Hosios (1990) condition is satisfied, due to random search and technology adoption. The decentralised solution has too many low technology vacancies because agents face a trade off between accepting the current offer and waiting for a better offer to come in next periods. When search is directed, the decentralised equilibrium is efficient even with two-sided heterogeneity, as in Shimer (2005) and Shi (2002), unless search costs are positive, as in Acemoglu and Shimer (2000): in that case, efficient allocations can be implemented subsidising search. In the model presented here, misallocations result from equal opportunity and discrimination. Furthermore, every worker sends out one application. As a result, subsidising search or unemployment benefits would not help.

The effect of minimum wages could be instead counterproductive if there is an equal opportunity legislation. Suppose that we are in the situation where only high technology vacancies are posted: in that case, increasing w_b^H would induce a workers to apply to that wage, reducing consistently b workers' market wage. If instead low technology jobs are posted, a minimum wage slightly above w_b^L could increase slightly b workers' wage, but, because of free entry, substantially increasing unemployment and finally reducing market wage. Furthermore, if the minimum wage gets too high, firms cannot dissuade a workers to apply to low technology jobs. Hence, such a high minimum wage would imply a dramatic reduction in b workers' market wage.

Overall, removing equal opportunities legislations turns out to be the most efficient policy to improve the labour outcomes of discriminated workers: in such case, expected production would be as in an economy without discrimination, and this would allow to collect revenue to implement policies targeted at disadvantaged workers, such as subsidising their education.

5 Conclusions

I proposed a wage-posting model of a labour market where firms first decide which technology to adopt and then use a discriminatory hiring rule to choose among equally productive applicants. Firms might not choose the most productive technology, and, when this happens, discriminated workers face lower wages and higher unemployment rates. Hence, both theoretical and empirical studies aiming to detect and prevent discrimination should take into account the endogeneity of the matching profile.

In such a framework, the suboptimality of investments disappears once firms are allowed to discriminate through wages. As a result, removing equal opportunities legislations turns out to be the most efficient policy to improve the labour outcomes of discriminated workers.

Occupational segregation could be explained in alternative ways and looking solely at wages does not provide conclusive evidence on the existence of discrimination. I showed though that differentials in capital intensities of firms that employ workers of minority groups can serve as a test to detect discriminatory practices. If occupational segregation were due to skill complementarities or hiring through referrals, there would be no reason to observe any systematic difference in capital intensity across firms that employ minority workers.

There are many interesting directions in which the model proposed here could be extended. Let me discuss two. First, introducing cultural and linguistic complementarities in the model could explain why occupational segregation happens at different levels for different groups of workers as showed by Bayard et al. (2003) and Hellerstein and Neumark (2008). Second, I have assumed that firms are able to commit on wages. Nonetheless, if some workers cannot operate the high technology, firms have incentives to post bargaining rules, as in Michelacci and Suárez (2006). Since heterogeneity in productivity is typically more relevant for more skilled workers, this would generate that the wage gap is less relevant for them but the unemployment gap is still strong, as documented for example in Lang and Lehmann (forthcoming) or Bjerk (2007). Conversely, for less skilled workers, since their productivity gaps are small, firms would still post wages: an equilibrium with positive wage and unemployment gaps would hence hold.

Appendix A

Proof of Theorem 1. I first state and prove the following Lemmata.

Lemma 1 *In equilibrium, if a firm posting w^J attracts workers ($q(w^J) > 0$), then*

$$Y(J)/V = e^{q(w^J)} \qquad \textit{and} \qquad \pi(w^J) = \beta(q(w^J))Y(J) - K^J.$$

Proof of Lemma 1. From (2), $\pi^J = (1 - e^{-q(w^J)})Y(J) - q(w^J)V$. Taking the first order conditions of this profit function with respect to the queue, one obtains $V = Y(J)e^{-q(w^J)}$, which, if substituted back into the profit function to get the profits, yields exactly $\pi(w^J) = \beta(q(w^J))Y(J) - K^J$. ■

From the previous Lemma, all firms face the same problem. Hence, they will post the same wage.

Lemma 2 *In equilibrium, $H^* = 1$.*

Proof of Lemma 2. Suppose first $H^* \in (0, 1)$. By Lemma 1, this would imply $V/y = \theta e^{-q(w^H)}$ and $V/y = e^{-q(w^L)}$. Hence, $q(w^H) = q(w^L) + \ln\theta > q(w^L)$. By the zero profit condition, $\beta(q(w^L)) = K^L/y$ and $\beta(q(w^H)) = K^H/\theta y$. Under the assumptions on the production technology $K^H/\theta y < K^L/y$. Since $\beta' > 0$, this implies $q(w^H) < q(w^L)$, a contradiction. Suppose now that $H^* = 0$. Then, $V/y = e^{-q(w^L)}$ and $\pi(w^J) = \beta(q(w^J))Y(J) - K^J$ applies for all the firms. But it's easy to see that the deviation to post a high technology vacancy to be offered to a worker for a slightly higher wage is a profit improving deviation for the firm. ■

Since all workers apply to high technology firms and they all post the same wage, f^* is defined by $f^{nd}(w^{nd}) = 1$. Queues are given by $q(w^{nd}) = N/M$. Since in equilibrium all firms make the same profits and all workers obtain the same expected wage, (5) and (6) result. ■

Proof of Proposition 1. The result is established in steps. First, I establish that there is no equilibrium wage to which both types of workers apply. If this were the case, (9) should hold for both types of workers. By substituting these into the profit function (10), the problem of a firm receiving both types of applicants is

$$\max_{w^H}(1-e^{-q_a(w^H)})(Y(H)-w^H)+e^{-q_a(w^H)}(1-e^{-q_b(w^H)})(Y(H)-w^H).$$

The partial derivative of the profits with respect to the wage is:

$$\frac{\partial \pi^H}{\partial w^H}=-\left(1-e^{-q_a(w^H)-q_b(w^H)}\right)+e^{-q_a(w^H)-q_b(w^H)}(Y(H)-w^H)\left[\frac{\partial q_a(w^H)}{\partial w^H}+\frac{\partial q_b(w^H)}{\partial w^H}\right].$$

The first term of the expression is clearly negative, while the second depends on the sign of $\partial q_a(w^H)/\partial w^H + \partial q_b(w^H)/\partial w^H$.

Since (9) is satisfied for a workers,

$$\frac{\partial w^J}{\partial q_a(w^H)}=\frac{V_a[1-e^{-q_a(w^H)}-q_a(w^H)e^{-q_a(w^H)}]}{(1-e^{-q_a(w^H)})^2}$$

which is different from 0. Given $x \in \mathbb{R}$, let $k(x)$ be $x(e^x-1)/[2(e^x-x-1)]$. Hence,

$$\begin{aligned}\frac{\partial q_a(w^H)}{\partial w^H} &= \frac{(1-e^{-q_a(w^H)})^2}{V_a[1-e^{-q_a(w^H)}-q_a(w^H)e^{-q_a(w^H)}]}= \\ &= \frac{2}{w^H}k(q_a(w^H)).\end{aligned}$$

Similarly, using (9) and differentiating on both sides by w^H

$$1=\frac{e^{q_a(w^H)}V_b[1-e^{-q_b(w^H)}-q_b(w^H)e^{-q_b(w^H)}]}{(1-e^{-q_b(w^H)})^2}\frac{\partial q_b(w^H)}{\partial w^H}+\frac{V_b q_b(w^H)}{1-e^{-q_b(w^H)}}e^{q_a(w^H)}\frac{\partial q_a(w^H)}{\partial w^H},$$

which yields

$$\frac{\partial q_b(w^H)}{\partial w^H}=\frac{e^{-q_a(w^H)}(1-e^{-q_b(w^H)})^2}{V_b[1-e^{-q_b(w^H)}-q_b(w^H)e^{-q_b(w^H)}]}\left(1-\frac{V_b}{V_a}\frac{q_b(w^H)}{1-e^{-q_b(w^H)}}\frac{e^{q_a(w^H)}(1-e^{-q_a(w^H)})^2}{[1-e^{-q_a(w^H)}-q_a(w^H)e^{-q_a(w^H)}]}\right).$$

Given $x \in \mathbb{R}$, let $g(x)$ be $2(xe^x - e^x + 1)/[x(e^x - 1)]$. After some algebra,

$$\begin{aligned}
\frac{\partial q_b(w^H)}{\partial w^H} &= \underbrace{\frac{1}{V_b}}_{>0} \underbrace{\frac{(1-e^{-q_b(w^H)})^2}{[1-e^{-q_b(w^H)} - q_b(w^H)e^{-q_b(w^H)}]}}_{>0} \underbrace{\frac{e^{-q_a(w^H)}(1-e^{-q_a(w^H)} - q_a(w^H))}{[1-e^{-q_a(w^H)} - q_a(w^H)e^{-q_a(w^H)}]}}_{<0} = \\
&= -\frac{2}{w^H} k(q_a(w^H))k(q_b(w^H))g(q_a(w^H)).
\end{aligned}$$

The first expression is positive and well defined as long as there is a non-zero measure of b workers. The second expression is positive, decreasing and approaching 1 at infinity. After rearranging,

$$\frac{\partial q_a(w^H)}{\partial w^H} + \frac{\partial q_b(w^H)}{\partial w^H} = -\frac{2}{w^H} k(q_a(w^H))[k(q_b(w^H))g(q_a(w^H)) - 1],$$

which is negative for all $q_a(w^H) > 0$ and $q_b(w^H) > 0$. Hence, a wage to which both workers apply cannot be part of an equilibrium

Second, I characterise the problem of firms attracting only one type of worker. The problem of firms attracting a workers is the following:

$$\max_{q_a(w^H)>0} (1 - e^{-q_a(w^H)})Y(H) - q_a(w^H)V_a, \tag{A-1}$$

subject to the condition that b workers are not willing to apply to wage w_a^H that attracts a workers in sector H, that is, $V_b \geq e^{-q_a(w_a^H)(w_a^H)} w_a^H$.

The problem yields a solution given by

$$\begin{aligned}
Y(H)e^{-q_a(w_a^H)} - V_a &= 0 \text{ if } V_b^* > e^{-q_a(w_a^H)} w_a^H \\
Y(H)e^{-q_a(w_a^H)} - V_a &\leq 0 \text{ if } V_b^* = e^{-q_a(w_a^H)} w_a^H.
\end{aligned}$$

Since $e^{-q_a(w_a^H)}$ is strictly decreasing in the queue, the problem yields a unique wage which is increasing in the queue. All firms in this category will choose the same wage, w_a^H as in (11), which gives rise to a market utility as in (13).When there are no a workers, the problem is symmetric to the case where only a workers apply, and it is the following:

$$\max_{q_b(w^H)>0}(1-e^{-q_b(w^H)})Y(H)-q_b(w^H)V_b,$$

subject to the condition that a workers are not willing to apply to the wage w_b^H posted in sector J to attract b workers, i.e., $V_a \geq w_b^H$, which implies that $w_a^H \geq w_b^H$.

Again, all firms will choose the same wage to attract only b workers, w_b^H. The problem yields a solution given by

$$\begin{aligned} Y(H)e^{-q_b(w_b^H)}-V_b &= 0 \text{ if } V_a > w_b^H, \\ Y(H)e^{-q_b(w_b^H)}-V_b &\geq 0 \text{ if } V_a = w_b^H. \end{aligned}$$

Since $e^{-q_b(w_b^H)}$ is strictly decreasing in the queue, the problem yields a unique wage that is increasing in the queue. Using this first order condition, it is easy to obtain wages as in (12) and market utility as in (14). ■

Proof of Remark 2. By Remark 1, $H_a^* \in (0,1)$. By Proposition 1, there are firms that post wages to attract a workers only. Hence, $V_a/y = \theta e^{-q_a(w_a^H)}$. In the low technology sector two cases might apply. (i) a workers apply to wages that do not attract b workers; in such case, the result follows immediately by Lemma 2. (ii) a workers apply to wages where b workers apply as well. In such case, $V_a = v_b = ye^{-q(w_a^L)-q(w_b^L)}$. Hence, $q(w_a^H) = q(w_a^L)+q(w_b^L)+\ln\theta > q(w_a^L)+q(w_b^L)$. Profits are given by $\beta(q(w_a^L)+q(w_b^L)) = K^L/y$ and $\beta(q(w_a^H)) = K^H/\theta y$. But $K^H/\theta y < K^L/y$. Since $\beta' > 0$, this implies $q(w_a^H) < q(w_a^L)+q(w_b^L)$, a contradiction. ■

Proof of Theorem 2. When b workers are employed in the L sector, the maximization problem of firms attracting a workers is unaffected since these workers do not face the competition of any other agent in the economy. Hence, as in the other equilibria, (13) and (11) hold. The definition of the queue for a workers, q_a^*, implies (15).

In the low sector, only b are applying. Hence, the problem of L firms is:

$$\max_{q_b(w_b^L)>0} (1 - e^{-q_b(w_b^L)})y - q_b(w_b^L)V_b,$$

which leads to the first order condition $ye^{-q_b(w_b^*)} = V_b^*$.

Profits of firms posting vacancies in the low sector are obtained substituting back the first order conditions into the profits function (10), so that (16) is obtained. The two zero profits conditions in the two sectors determine H_a^* and M^*. Using (15) and (16), (17) and (18) are obtained.

The conditions (15), (16), (17) and (18) define an equilibrium if both firms and agents have no profitable deviations. First of all, b workers should not be willing to apply to the wage offered to a applicants in the high sector. Since all a workers apply in that sector, a b worker will be chosen only with probability $e^{-q_a^H*}$. Hence, the condition is $V_b^* > e^{-q_a^H*}w_a^*$.

On the other hand, firms do not have to find profitable to offer a wage to b workers in the high technology sector. This is true whenever $w_b^* \geq V_a^*$, which leads to condition (22).

Since $V_b^* = \mu_b^* w_b^*$, (22) implies $V_b^* > e^{-q_a^H*}w_a^*$ and I concentrate on the first. Since the left hand side of this expression is continuous and strictly increasing in θ, there exists a $\bar{\theta}(y, K^H, K^L)$ such that (22) is satisfied for all $\theta < \bar{\theta}(y, K^H, K^L)$.

Finally, in equilibrium a workers should not be willing to apply for a position in the low technology sector. This is true whenever $V_a^* > w_b^*[x^L + (1 - x^L)e^{-B\left(K^L/y\right)}]$. This implies (23).

When b workers are employed in the H sector, queues are as follows:

$$q_a(w_a^H) = \frac{\alpha N}{H_a^{**}M^{**}},$$

$$q_b^{**} = \frac{(1-\alpha)N}{(1-H_a^{**})M^{**}}.$$

Since all firms need to get zero profits in equilibrium,

$$\frac{\alpha N}{H_a^{**}M^{**}} = B\left(\frac{K^H}{\theta y}\right),$$

$$\frac{\alpha N}{M^{**}H_a^{**}}e^{-\frac{\alpha N}{M^{**}H_a^{**}}} = (1-e^{-\frac{\alpha N}{H_a^{**}M^{**}}})e^{-\frac{(1-\alpha)N}{M^{**}H_b^{**}}}.$$

Defining $C\left(K^H/\theta y\right) = \log(e^{B\left(K^H/\theta y\right)} - 1) - \log B\left(K^H/\theta y\right)$, (15) and (19) are obtained.

Hence, substituting these expressions in the definitions of the queues in this equilibrium, one can obtain the proportion of firms posting vacancies in each sector of the economy as in (20) and the total measure of firms as in (21). This concludes the proof of Theorem 2. ■

Proof of Proposition 2. Production, denoted by $EY(K^L, K^H, N, \alpha)$, is defined as follows:

$$EY(K^L, K^H, N, \alpha) = \sum_{J=H,L}\int[(1-e^{-q_a(w^J)-q_b(w^J)})Y(J) - K^J]df^*(w^J)$$

Using the equilibrium queues and distribution of wage offers, it follows that

$$EY^* = N\alpha V_a^* + N(1-\alpha)V_b^*,$$

$$EY^{**} = N\alpha V_a^* + N(1-\alpha)V_b^{**}.$$

Hence, production is higher when all firms post high technology vacancies. ■

Proof of Corollary 1. Let $0 < \epsilon < \theta - 1$ be the productivity gap in the H sector between a and b workers. Hence, the profits of a firm in the high technology sector are

$$\begin{aligned}\pi^H(w^H) &= \left(1-e^{-q_a(w^H)}\right)\left[x^H + (1-x^H)e^{-q_b(w^H)}\right]\theta y \\ &\quad + \left(1-e^{-q_b(w^H)}\right)\left[1-x^H + x^H e^{-q_a(w^H)}\right](\theta y - \epsilon) - q_a(w^H)V_a - q_b(w^H)V_b.\end{aligned}$$

Since $\partial\pi^H/\partial x^H = 0$, it is optimal to set $x^H = 1$. Given this, Proposition 1 applies. But now the queue of b workers in the H sector would be

$$C'\left(K^H/\theta y, \epsilon\right) = \log\left[\frac{\theta y - \epsilon}{\theta y}e^{B\left(K^H/\theta y\right)} - 1\right] - \log\left[B\left(K^H/\theta y\right) - \frac{\theta y - \epsilon}{\theta y}e^{B\left(K^H/\theta y\right)}\right],$$

instead of the function $C\left(K^H/\theta y\right)$ defined above. If $\epsilon \to 0$, the two functions coincide. Hence, the threshold θ that determines whether it is not to offer a H job to a b worker when L positions are posted is

$$e^{-B\left(\frac{K^L}{y}\right)} > \frac{1-e^{-C'\left(K^H/\theta y,\epsilon\right)}}{C'\left(K^H/\theta y,\epsilon\right)}\theta e^{-B\left(\frac{K^H}{\theta y}\right)}.$$

It is easy to see that there is a $\theta \leq \hat{\theta}(y, K^H, K^L, \epsilon) < \bar{\theta}(y, K^H, K^L)$ such that this condition holds. Finally, the result on x^L follows from the fact that, when $\theta \leq \hat{\theta}(y, K^H, K^L, \epsilon)$, posting a selection rule x^L high enough to attract a workers to the posted wage would obviously not be profitable, since this would reduce the queue of b workers more than attract a workers, given the difference in market utilities and given that everybody is equally productive in the L sector, reducing profits. ■

Proof of Proposition 3. When forms post different wages to attract both workers, the maximization problem of a firm is

$$\max_{w_a^H, w_b^H}(1-e^{-q_a(w_a^H)})[x+(1-x)e^{-q_b(w_b^H)}](Y(H)-w^H)+[1-x+xe^{-q_a(w_a^H)}](1-e^{-q_b(w_b^H)})(Y(H)-w_b^H),$$

where $x=1$ if $w_a^H \leq w_b^H$ and $x=1$ otherwise. In other words, firms apply a discriminatory hiring rule when wages for workers are the same, otherwise they prefer the cheaper type.

After substituting for wages, the problem of the firm becomes

$$\max_{w_a^H, w_b^H}\left[1-e^{-q_a(w_a^H)-q_b(w_b^H)}\right]Y(H)-V_a q_a(w_a^H)-V_b q_b(w_b^H).$$

The first order conditions of this problem are $Y(J)e^{-q_a(w_a^H)-q_b(w_b^H)} = \widetilde{V}_a = \widetilde{V}_b$. This directly implies that the result of Lemma 2 would go through for both workers' types. Using the zero profits conditions, $\widetilde{q}_a(w_a^H)+\widetilde{q}_b(w_b^H) = B\left(K^H/\theta y\right)$. Since all workers apply to all firms with the same probability, using the definitions of queues, $\alpha\widetilde{q}_a = (1-\alpha)\widetilde{q}_b$. Hence, (24) and (25) follow. Substituting these back in the definition of queues, (26) results. ■

References

Acemoglu, Daron, (1998), "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *The Quarterly Journal of Economics*, *113* (4), pp. 1055–1089.

—, (1999), "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *The American Economic Review*, *89* (5), 1259–1278.

—, (2001), "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, *19* (1), 1–21.

— and Robert Shimer, (2000), "Wage and Technology Dispersion," *The Review of Economic Studies*, *67* (4), 585–607.

Albrecht, James, Anders Björklund, and Susan Vroman, (2003), "Is There a Glass Ceiling in Sweden?," *Journal of Labor Economics*, *21* (1), pp. 145–177.

—, Pieter A. Gautier, and Susan Vroman, (2006), "Equilibrium Directed Search with Multiple Applications," *Review of Economic Studies*, *73* (4), 869–891.

Altonji, Joseph G. and Rebecca M. Blank, (1999), "Race and gender in the labor market," in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Elsevier, pp. 3143–3259.

Anderson, Lisa R., Roland G. Fryer, and Charles A. Holt, (2005), "Discrimination: Experimental Evidence from Psychology and Economics," in William M. Rodgers, ed., *William M. Rodgers, ed.*, Handbook on Economics of Discrimination, Edward Elgar.

Azmat, Ghazala, Maia Güell, and Alan Manning, (2006), "Gender Gaps in Unemployment Rates in OECD Countries," *Journal of Labor Economics*, *24* (1), 1–38.

Bayard, Kimberly, Judith Hellerstein, David Neumark, and Kenneth Troske, (2003), "New Evidence on Sex Segregation and Sex Differences in Wages from Matched Employee-Employer Data," *Journal of Labor Economics*, *21* (4), 887–922.

Bjerk, David, (2007), "The Differing Nature of Black-White Wage Inequality Across Occupational Sectors," *J. Human Resources*, *XLII* (2), 398–434.

Black, Dan, Amelia Haviland, Seth Sanders, and Lowell Taylor, (2006), "Why Do Minority Men Earn Less? A Study of Wage Differentials among the Highly Educated," *Review of Economics and Statistics*, *88* (2), 300–313.

Bowlus, Audra J. and Zvi Eckstein, (2002), "Discrimination and Skill Differences in an Equilibrium Search Model," *International Economic Review*, *43* (4), 1309–1345.

Burdett, Kenneth, Shouyong Shi, and Randall Wright, (2001), "Pricing and Matching with Frictions," *Journal of Political Economy*, *109* (5), 1060–1085.

Duleep, Harriet Ortcutt and Seth Sanders, (1992), "Discrimination at the Top: American-Born Asian and White Men," *Industrial Relations: A Journal of Economy and Society*, *31* (3), 416–432.

Fairlie, Robert W. and William A. Sundstrom, (1997), "The Racial Unemployment Gap in Long-Run Perspective," *The American Economic Review*, *87* (2), pp. 306–310.

Filippin, Antonio and Francesco Guala, (2011), "Costless Discrimination and Unequal Achievements in a Labour Market Experiment," *IZA Discussion Paper No. 6187.*

Flabbi, Luca, (2010), "Gender Discrimination Estimation In A Search Model With Matching And Bargaining," *International Economic Review*, *51* (3), 745–783.

Frank, Jeff, (2006), "Gay Glass Ceilings," *Economica*, *73* (291), 485–508.

Galenianos, Manolis and Philipp Kircher, (2009), "Directed search with multiple job applications," *Journal of Economic Theory*, *144* (2), 445–471.

Gardeazabal, Javier and Arantza Ugidos, (2005), "Gender wage discrimination at quantiles," *Journal of Population Economics*, *18* (1), 165–179.

Grodsky, Eric and Devah Pager, (2001), "The Structure of Disadvantage: Individual and Occupational Determinants of the Black-White Wage Gap," *American Sociological Review*, *66* (4), 542–567.

Hellerstein, Judith K. and David Neumark, (2008), "Workplace Segregation in the United States: Race, Ethnicity, and Skill," *The Review of Economics and Statistics*, *90* (3), 459–477.

Heywood, John S. and Daniel Parent, (2012), "Performance Pay and the White-Black Wage Gap," *Journal of Labor Economics*, *30* (2), 249–90.

Hosios, Arthur J, (1990), "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, *57* (2), 279–98.

Kaas, Leo, (2009), "Does Equal Pay Legislation Reduce Labour Market Inequality?," *Scandinavian Journal of Economics*, *111* (1), 51–71.

Kimberly, Judith Hellerstein David Neumark Bayard and Kenneth Troske, (1999), "Why Are Racial and Ethnic Wage Gaps Larger for Men than for Women? Exploring the Role of Segregation Using the New Worker-Establishment Characteristics Database," in Spletzer Theeuwes Haltiwanger, Lane and Kenneth Troske, eds., *The Creation and Analysis of Employer-Employee Matched Data*, Amsterdam: Elsevier Science B.V., pp. 175–203.

Lang, Kevin and Jee-Yeon K. Lehmann, (forthcoming), "Racial Discrimination in the Labor Market: Theory and Empirics," *Journal of Economic Literature.*

_ and Michael Manove, (2011), "Education and Labor Market Discrimination," *American Economic Review*, *4* (101), 1467–96.

_ , _ , and William T. Dickens, (2005), "Racial Discrimination in Labor Markets with Posted Wage Offers," *American Economic Review*, *95* (4), 1327–1340.

Méon, Pierre-Guillaume and Ariane Szafarz, (2011), "The modern corporation as a safe haven for taste-based discrimination: An agency model of hiring decisions," *Labour Economics*, *18* (4), 487–497.

Michelacci, Claudio and Javier Suárez, (2006), "Incomplete Wage Posting," *Journal of Political Economy*, *114* (6), 1098–1123.

Pendakur, Krishna and Simon Woodcock, (2010), "Glass Ceilings or Glass Doors? Wage Disparity Within and Between Firms," *Journal of Business Economic Statistics*, *28* (1), 181–189.

Pudney, Stephen and Michael A. Shields, (2000), "Gender and Racial Discrimination in Pay and Promotion for NHS Nurses," *Oxford Bulletin of Economics and Statistics*, *62*, 801–835.

Ritter, Joseph A and Lowell J Taylor, (2011), "Racial Disparity in Unemployment," *Review of Economics and Statistics*, *93* (1), 30–42.

Rosen, Asa, (1997), "An equilibrium search-matching model of discrimination," *European Economic Review*, *41* (8), 1589–1613.

Shi, Shouyong, (2002), "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Based Technology," *Review of Economic Studies*, *69* (2), 467–91.

_ , (2006), "Wage differentials, discrimination and efficiency," *European Economic Review*, *50* (4), 849–875.

Shimer, Robert, (2005), "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, *113* (5), 996–1025.

Weinberger, Catherine J., (1998), "Race and Gender Wage Gaps in the Market for Recent College Graduates," *Industrial Relations: A Journal of Economy and Society*, *37* (1), 67–84.

Winter-Ebmer, Rudolf and Josef Zweimüller, (1997), "Unequal Assignment and Unequal Promotion in Job Ladders," *Journal of Labor Economics*, *15* (1), pp. 43–71.