Efficient Sorting in Frictional Labor Markets with Two-Sided Heterogeneity

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Abstract

This paper studies how search externalities and wage bargaining distort vacancy creation and the allocation of workers to jobs in markets with two-sided heterogeneity. To do so, I propose a model of a frictional labor market where heterogeneous workers decide which job to look for and firms decide which technology to adopt. In equilibrium, there is perfect segmentation across sectors, which is determined by a unique threshold of workers' productivity. This threshold is inefficient due to participation and composition externalities. The Pigouvian tax scheme that decentralizes optimal sorting shows that these externalities have opposite signs. Furthermore, their relative strength depends on the distribution of workers' skills, so that when there are many (few) skilled workers, too many (few) high technology jobs are created.

JEL Codes: J21; J31; J64.

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1 Introduction

Search models with two-sided heterogeneity have been used to formulate most of the theories that relate recent changes in unemployment and wage inequality to skill biased technological change.[1] This success is due to the fact that, *(i)*, search models provide a parsimonious and flexible tool to study unemployment, and, *(ii)*, heterogeneity is necessary to embody skill biased technological change.

Nonetheless, there is not yet a coherent characterization of efficient sorting for search markets with heterogeneous agents. While it is understood that decentralized equilibria are usually inefficient, it is not clear whether good workers hurt bad workers, usually by inducing the creation of too many high technology jobs, or rather the other way around.[2] However, this is crucial to understand the nature of the inefficiencies involved and, eventually, to design policies to alleviate them.

The aim of this paper is precisely to understand how search and wage bargaining distort sorting in markets with heterogeneous agents. To do so, I propose a tractable model with heterogeneous workers who are able to direct their search to their preferred type of job, and firms that decide which technology to adopt. Usually, two-sided heterogeneity leads to great complications in models whose main attractiveness is parsimony: the set of equilibria explodes, and multiple matching profiles can be sustained in equilibrium. The model I propose instead has a unique equilibrium that displays perfect segmentation across sub-markets. This feature makes the social welfare analysis possible and rich enough to uncover the forces behind the discrepancy between decentralized equilibrium and optimal sorting.

In the model, the average productivity of a sector depends on the skills composition of job seekers. Hence, workers' search behavior generates externalities that do not vanish even when the Hosios (1990) condition is satisfied. Most importantly, the sign of these externalities depends on the distribution of workers' skills. In particular, if there are many high skilled workers, the average productivity in high technology jobs is very high. As a result, the labor market conditions in that segment are so good that more workers than it is socially desirable look for a job there. On the contrary, when there are many low skilled workers, the planner would like to subsidize search in the high technology sector where the externality due to workers not searching

[1] Some examples are Acemoglu (1999, 2001); Albrecht and Vroman (2002); Dolado, Jansen and Jimeno (2009); Regev and Zoabi (in press); Shi (2002).

[2] While in Albrecht, Navarro and Vroman (2010), Blázquez and Jansen (2008) and Lang, Manove and Dickens (2005) skilled workers hurt bad workers, the opposite is true in Charlot, Malherbet and Ulus (2013) and Moen (2003).

hard enough is stronger. While this externality is standard in search models, when workers have different productivity it affects the segmentation threshold, and it is present for all levels of workers' bargaining power. Hence, while there is scope for policy intervention to reduce inefficiencies, the direction of such intervention depends on the supply of skills.

Beyond the policy implications, this analysis points out that, if we want to understand the properties of decentralized equilibria in search models with two-sided heterogeneity, it is important to consider endogenous (and non trivial) participation decisions in the labor market. Indeed, when agents are heterogeneous, the value of being in a particular segment of the market depends on which types look for a job in each sector, so that composition and participation externalities emerge.

The modeling framework is introduced in detail in Section 2. There is a continuum of workers' types, and *ex ante* homogeneous firms that open vacancies by choosing between two technologies: a traditional technology, where all workers have the same productivity, and an advanced technology, where more skilled workers are more productive. Hence, while workers are heterogeneous *ex ante*, firms are heterogeneous only *ex post*, and the proportion of posted vacancies of each type is endogenous.

The two types of jobs are posted in separated islands, and, as in Moen (1997), workers have to decide in which island of the economy to direct their search, knowing that in both the matching technology is random. However, differently from Moen (1997), wages are set through Nash bargaining.

Firms post vacancies of each type until the expected profits are driven to zero by free entry, while workers look for a job in the sector where the expected value of search is higher. Since the productivity is, *(i)*, constant in workers' types in the low technology sector, and, *(ii)*, increasing in the high technology sector, there is a unique threshold above which all workers look for high technology jobs. Such threshold, which depends on the relative labor market conditions in the two sectors, is enough to fully describe the allocation of workers to firms. Hence, while in standard search models with two-sided heterogeneity worker-firm allocations are fixed or change abruptly,[3] in my model they change continuously. As a result, there is an interesting choice for the "marginal" worker type employed in the low technology sector.

This feature allows studying sorting in a meaningful way by comparing the result of the decentralized equilibrium with the social optimum, which I derive in Section 3. In a framework where firms can commit on posted wages conditional on workers' types, or segmentation is exogenous, the behavior of workers of a

[3] For example, see Albrecht and Vroman (2002) or Shi (2002).

type does not create externalities on the matching opportunities of the other types. In my model, however, the labor market tightness in the high technology sector depends on the pool of workers participating in that sector. As a result, the segmentation threshold is not efficient.

Since the interaction between workers and firms are complex, in Section 4 I study the Pigouvian tax scheme that decentralizes the optimal allocation. This allows to better understand the nature of the inefficiency.

In particular, in addition to the usual entry, thick-market and congestion externalities of search models, other distortions emerge. First, there are (negative) *composition externalities*: workers do not take into account the effect of their search behavior on the average productivity in the high technology sector, which lowers firms' expected value of opening a vacancy and other workers' expected value of search. Then, there are *participation externalities* due to the fact that workers' search behavior depends on unemployment benefits rather than average productivity. This (positive) externality is hence stronger in the high technology sector.

As a result, both the direction and extent of the overall search externalities depends on the distribution of workers' skills, even when the Hosios (1990) condition is satisfied. The intuition is the following. When there are many skilled workers, posting a high technology vacancy is very profitable because realized matches generate on average a high surplus. In other words, the composition externality is very strong. This implies that the labor market tightness in the high technology sector is high, so that a lot of workers look for a job there. However, in making this decision, workers do not take into account the impact of their choice on the expected productivity of advanced firms. As a result, the social planner would like the segmentation threshold to be higher to increase the expected productivity in the high technology sector. On the contrary, when there are very few skilled workers, while such composition externality is weak, the participation externality is strong, and more so in the high technology sector. As a result, there are too few high technology vacancies. Hence, the social planner would like to subsidize the most skilled workers that are looking for low technology jobs to induce them to participate in the high technology sector instead.

To assess the separate role of frictions and the wage determination protocol, I compare the decentralized equilibrium with the one of a framework where I abstract from the inefficiencies coming from wage bargaining. In that case, sorting is still inefficient because of the composition externality, and too many high technology jobs are created irrespectively of the skill distribution.

Overall, my model allows understanding how search externalities and wage bargaining distort vacancy creation and the allocation of workers to jobs, possibly guiding government intervention. In particular, only sector and type specific taxes or unemployment benefits can solve the distortions that emerge due to endogenous segmentation. Hence, other policies such as minimum wages, sector independent tax rates, or centralized bargaining over wages cannot decentralize the optimal allocation; rather, in some cases, they create additional distortions.

Literature Review. This paper is related to the literature of two-sided heterogeneity in random search models, a framework that is widely used to assess the effects of skill biased technological change. One of the seminal papers in this literature is Albrecht and Vroman (2002), in which they study an economy with two types of workers and a production technology such that unskilled workers are not employable in high technology jobs. Both a fully segmented and a partially segmented equilibrium can be sustained. Blázquez and Jansen (2008) and Uren (2006) study the efficiency properties of such framework, and find that, when the market segments, the Hosios (1990) rule is enough to ensure efficiency. This is not the case here because the segmentation threshold is itself endogenous, a feature that leaves room for efficiency to be violated. Gautier (2002) and Khalifa (2012) modified Albrecht and Vroman (2002) framework by introducing search on the job and the fact that unskilled workers can apply to complex jobs. In such framework, unskilled workers might benefit from the presence of skilled workers who have a high productivity at simple jobs.

Recently, several papers have studied efficiency in search model with wage bargaining, heterogeneous agents, and meaningful participation choices. For example, Albrecht et al. (2010) consider a one-period model of labor market participation with a distribution of workers' productivity: in that framework, there is always excessive (high technology) vacancy creation. Charlot et al. (2013) study an economy with a frictionless informal sector where productivity is given by a distribution. They find that the informal sector is too big, but constrained efficiency is achieved when the Hosios (1990) rule is satisfied and the interest rate goes to zero. In this paper, I extend both findings by providing a comprehensive and coherent set of results in a fully dynamic model where all segments of the labor market display some frictions. This more general framework allows me understanding which role search frictions, wage determination and dynamics play in the different predictions of those models.

Teulings and Gautier (2004) study an approximation of the equilibrium that holds close to the Walrasian outcome in an economy with a continuum of workers' and firms' types. They show that search in the decentralized equilibrium is too choosy. The model proposed here is complementary to that paper because of two reasons. First, they consider an economy with differentiated goods, and their approximation does not hold in the homogeneous goods case studied here. Second, they assume a quadratic matching technology, which displays no congestion externalities, while I use a standard constant return matching technology. This is the most interesting case in which to study efficiency because, theoretically, Diamond (1982) showed that with increasing returns to scale the decentralized equilibrium is inefficient even when agents are homogeneous, and, empirically, most studies support the hypothesis of constant returns to scale.[4]

Inefficiencies emerge also in frameworks where wages are set competitively if firms are not allowed to post type-contingent contracts (Shi, 2006). However, there is controversy with respect to the direction of such inefficiencies. While Lang and Dickens (1992) and Lang et al. (2005) find that less skilled workers suffer negative externalities from the presence of more skilled workers, Moen (2003) argues that the opposite could be true if productivity depends on a match specific component. In this paper, I contribute to this debate proposing a framework where both results are possible when wages are set via Nash bargaining, so that I can provide conditions under which “good” workers hurt “bad” workers.

Other authors have studied efficiency in search models with horizontal heterogeneity. For example, in Marimon and Zilibotti (1999) firms and workers are distributed around the Salop circle and the productivity of a match depends on the distance between the employer’s and employee’s type. Mismatch arises because agents face a trade off between productivity gains and the time needed to find a better partner; however, the Hosios (1990) condition ensures constrained efficiency. Decreuse (2008) shows that allowing for segmentation in that framework makes workers search on a too wide range of markets. This composition externality does not vanish for any level of workers’ bargaining power. Also Gautier, Teulings and Vuuren (2010) study efficiency allowing for search on the job and assuming monopsonistic wage setting. This paper is complementary to those because I focus on the vertical dimension of heterogeneity.

This paper is also related to the literature that studies the assignment of workers to jobs when there are frictions. Shimer and Smith (2000) study conditions for assortative matching in a framework where search is purely random, so that there can be multiple equilibria. Shimer and Smith (2001) consider a model with endogenous search intensity and show that the equilibrium is never efficient because bargaining distorts the marginal returns from search. In my model agents decide not only with whom to match, but also with whom they meet, so that markets endogenously segment, as in Jacquet and Tan (2007). In Eeckhout and Kircher (2010), as in this paper, sorting results from the role of prices (here wages) that indicate to agents where to look for a match. However, in their analysis search is fully directed, so that decentralized sorting is efficient. Overall, the assignment literature typically takes the number of agents of each type as exogenous and uses a production technology that is strictly increasing in agents’ types. With free entry of firms, this

[4]For a survey of the literature of the matching function, see Petrongolo and Pissarides (2001).

feature would give rise to a degenerate distribution of jobs, since only the most productive technology would be adopted, unless there are negligible skill differentials between workers with the same skill level and skills are more important in high technology jobs than in low technology jobs (Merlino, 2012). This paper instead aims at studying the interplay between job creation and technology adoption, so that I assume a skill-biased production technology.

2 The Model

In this section I present in detail the building blocks of the model, solve for the decentralized equilibrium, and derive some comparative statics.

Time. Time is continuous and $r \in (0, 1)$ is the interest rate used by both firms and workers to discount the future.

Workers. There is a continuum of workers of mass one, risk neutral and infinitely lived. Each worker is labeled by his skill level, or type, $x \in [\underline{x}, \infty)$, which is drawn according to a Pareto distribution function $f(x)$, which is defined as

$$f(x) = \begin{cases} \frac{\alpha \underline{x}^{\alpha}}{x^{\alpha+1}} & \text{if } x \geq \underline{x}, \\ 0 & \text{otherwise.} \end{cases}$$

In the following, I assume that $\alpha > 1$ and $\underline{x} = 1$. Let me denote the associated cumulative distribution by $F(x)$. The parameter α, the *tail* or *Pareto* index, defines the shape of the distribution: as α increases, very skilled workers become more scarce and there is less dispersion in the distribution of skills; when α tends to infinity, the distribution is degenerate at the skill level 1.

Let $u(x)$ be the proportion of unemployed workers of type x and let $\mathbf{u} = \int_1^{\infty} u(x) f(x) dx$ be the total unemployment rate, which is equal to the measure of unemployed workers, given that there is a mass one of them. Let $g(x) = u(x)/\mathbf{u}$ be the conditional distribution of unemployment. The distribution of unemployment is endogenous, and, in general, it is different from the distribution of types in the overall population.

Unemployed workers receive an instantaneous value of leisure $b > 0$, which can be interpreted as unemployment benefits.

Firms. There is a continuum of firms of mass m, risk neutral, infinitely lived, and each employing at most one worker;[5] m is determined endogenously by free entry. Each firm y can produce the unique homogeneous good of the economy in two possible ways. One is a traditional technology, labeled by L, which yields a production of $\underline{y}$ units of the good independently of the type of the worker. The interpretation is that this technology is easy to operate, but difficult to be improved, so that workers' skills do not matter. The other is an advanced technology, labeled by H, whose productivity depends linearly on the type of the worker. This technology is difficult to operate, so that more skilled workers are more productive. Given $\underline{y} > b$ and $\kappa > 0$, the production function can be written as follows:

$$Y(x,y) = \begin{cases} \underline{y} & \text{if } y = L, \\ \kappa x & \text{if } y = H. \end{cases}$$

Let v be the measure of positions posted (at a cost $c > 0$) and $\phi \in [0,1]$ be the proportion of low technology vacancies; both v and ϕ are endogenously determined in equilibrium. The number of filled vacancies in the labor market has to be equal to the number of employed workers, i.e. $1 - u = m - v$.

Jobs are either vacant or filled. Once jobs are filled, production takes place and wages are paid.

Wage Determination. Wages are determined through weighted Nash bargaining, where β is the share of surplus which goes to workers. I denote by $w(x,y)$ the wage paid by a firm adopting technology y, $y = L, H$, to a worker of type $x \in [1, \infty)$.

Matching Technology. Before meeting, firms are imperfectly informed about the location of workers' types. However, workers are able to direct their application to their preferred type of jobs. When a worker and a firm meet, the firm recognizes immediately the worker's type. Hence, a match is formed only if it is profitable for both parties. There is no search on the job. There is no recall of previous matches.

In each sub-market $y = L, H$, unemployed workers and vacancies meet according to a constant returns to scale matching function, $M(u_y, v_y)$. Hence, $M(u_y, v_y) = M(1, v_y/u_y)u_y = m(\theta_y)u_y$, where $\theta_y = v_y/u_y$ is the labor market tightness of the market segment y. Assume $M_u(u_y, v_y) > 0$, $M_v(u_y, v_y) > 0$, $\lim_{u \to 0} M(u_y, v_y) = 0$ and the usual Inada condition $\lim_{v_y \to 0} M(u_y, v_y) = 0$, that translates into $m'(\theta_y) > 0$, $\lim_{\theta_y \to \infty} m(\theta_y) = \infty$ and $\lim_{\theta_y \to 0} m(\theta_y) = 0$.

Job Separation. A worker-firm match is in place until hit by an exogenous idiosyncratic shock, which arrives

[5] Cahuc and Wasmer (2001) give conditions under which the assumption that each firm employs one worker is innocuous.

with probability $\delta \in (0,1)$.

Value Functions. Define $U(x)$ as the value of being unemployed for a worker of type x and $V(y)$ as the value of a vacancy of type y. The value $E(x,y)$ for worker x of a match with a firm of type y employing him at a wage $w(x,y)$ is given by:

$$E(x,y) = \frac{w(x,y) + \delta U(x)}{r+\delta}. \tag{1}$$

Similarly, the value $J(x,y)$ for firm y of the same match is given by:

$$J(x,y) = \frac{Y(x,y) - w(x,y) + \delta V(y)}{r+\delta}. \tag{2}$$

Meetings are consummated only if the joint surplus that would be generated by the match is non-negative; hence, a worker of type x and a firm of type y form a match if and only if $E(x,y) + J(x,y) \geq U(x) + V(y)$. Let A_L and A_H be the acceptance set of a firm of type L and H respectively, i.e. the set of types of workers they are willing to employ. The flow value for a firm of a posted vacancy of type y is given by

$$rV(y) = -c + \frac{m(\theta_y)}{\theta_y} \int_{A_y} [J(x,y) - V(y)] g(x) dx. \tag{3}$$

Workers show up to a vacancy according to the conditional distribution of unemployment $g(x)$, but a match is created only if the worker belongs to the acceptance set A_y. For a vacancy to be posted, it is necessary that

$$\frac{m(\theta_y)}{\theta_y} \int_{A_y} [J(x,y) - V(y)] g(x) dx \geq c,$$

but free entry of firms implies that in equilibrium $V(H) = V(L) = 0$. Hence, using (2) and (1), the acceptance sets can be written as:

$$A_L = \{x \in [0,1] : \underline{y} \geq rU(x)\}, \tag{4}$$

$$A_H = \{x \in [0,1] : \kappa x \geq rU(x)\}. \tag{5}$$

These expressions describe the two forces that determine sorting in the labor market: on the one hand, workers tend to be more attracted by the sector where they are more productive; on the other hand, they look at their value of unemployment, which depends on the labor market conditions.

The flow value of being unemployed for a worker of type x looking for a job in sub-market y, $y = L, H$, is:

$$rU_y(x) = b + m(\theta_y)\max\left\{E(x,y) - U(x), 0\right\}.$$

Nash bargaining implies that the wage associated with a match is given by the sum between the productivity of a worker and his value of being unemployed, weighted by the bargaining power, that is:

$$w(x,y) = \beta Y(x,y) + (1-\beta)rU(x). \tag{6}$$

This expression shows that wages are increasing in workers' value of unemployment, which in turn is increasing in the labor market tightness of the sector where the worker is looking for a job.

Segmentation. A worker of type x directs his search to sector y, for $y = L, H$, if he expects a higher value of being unemployed, $U_y(x)$, which, after some algebra, can be written as:

$$rU_y(x) = \frac{b(r+\delta) + \beta m(\theta_y)Y(x,y)}{r+\delta+\beta m(\theta_y)}.$$

Note that, given θ_L, the value of being unemployed is constant in the low technology sector, because production is constant in the worker's type x. On the contrary, given θ_H, the value of being unemployed in the high technology sector is increasing and continuous in x: since more skilled workers are more productive, their expected wages are higher. Note however that, because of the random matching technology, every worker in a sector faces the same job finding rate independently of his type.

The assumption $\underline{y} > b$ and the fact that $U_L(x)$ is constant in x while $U_H(x)$ is increasing in x ensure that there is a unique worker's type, denoted by s, which is indifferent between the two labor markets. This defines the threshold beyond which all workers apply for high technology jobs, while less skilled workers go to the market for low technology positions in such a way that in both sectors all matches are acceptable. More formally:

Lemma 1 *There exists a unique $s \in \mathbb{R}$ such that $rU_L(s) = rU_H(s)$. If $s < 1$, only high technology vacancies are posted.*

This cut-off rule is defined by

$$\frac{b(r+\delta)+\beta m(\theta_L)\underline{y}}{r+\delta+\beta m(\theta_L)}=\frac{b(r+\delta)+\beta m(\theta_H)\kappa s}{r+\delta+\beta m(\theta_H)}. \tag{7}$$

The threshold property assures segmentation, so that Proposition 1 follows immediately.

Proposition 1 *When an equilibrium in which both type of vacancies are posted exists, the labor market displays* ***complete segmentation****.*

Thus, I can now rewrite the acceptable sets as:

$$A_L = \{x \in [1,\infty] | 1 \leq x < \max\{1,s\}\},$$
$$A_H = \{x \in [1,\infty] | x \geq \max\{1,s\}\}.$$

Steady State. The steady state conditions ensure that the measure of workers who lose a job is the same as the measure of workers who find one:

$$\delta[1-u_L(x)] = u_L(x)m(\theta_L), \tag{8}$$
$$\delta[1-u_H(x)] = u_H(x)m(\theta_H). \tag{9}$$

Hence, for $j = L, H$,

$$u_j(x)=\frac{\delta}{m(\theta_j)+\delta}.$$

These conditions determine the distribution of unemployed workers in each market.

Equilibrium. Using the results above, the expected values of low technology vacancies can be rewritten as:

$$\begin{aligned} V(L) &= \frac{m(\theta_L)}{\theta_L}\frac{\int_1^s \left((1-\beta)\underline{y}-rU(x)\right)u(x)/u dF(x)}{r+\delta}-c \\ &= \frac{m(\theta_L)}{\theta_L}\frac{(1-\beta)(\underline{y}-rU_L)}{r+\delta}-c \\ &= \frac{m(\theta_L)}{\theta_L}\frac{(1-\beta)(\underline{y}-b)}{r+\delta+\beta m(\theta_L)}-c, \end{aligned} \tag{10}$$

which, given free entry of firms, implies the following remark.

Remark 1 *The labor market tightness in the low technology sector, θ_L, is increasing in the output $\underline{y}$, while it is decreasing in the cost of opening a vacancy c, in the unemployment benefit b, in workers' bargaining power β, in the interest rate r and in the job separation rate δ.*

Expression (10) neither depends on s nor on α: since the productivity of matches is constant independently of workers' type, neither the size of the market nor the distribution of skills influence the labor market tightness. Similarly, the expected value of high technology jobs can be written as:

$$\begin{aligned} V(H) &= \frac{m(\theta_H)}{\theta_H} \frac{(1-\beta)\int_s^\infty (\kappa x - rU(x))\, u(x)/u dF(x)}{r+\delta} - c \\ &= \frac{m(\theta_H)}{\theta_H} \frac{(1-\beta)}{r+\delta} \int_s^\infty \frac{(r+\delta)(\kappa x - b)}{r+\delta+\beta m(\theta_H)} \frac{u(x)}{\int_{A_H} u(x) dF(x)} dF(x) - c \\ &= \frac{m(\theta_H)}{\theta_H} \frac{(1-\beta)}{r+\delta+\beta m(\theta_H)} \left[\kappa \frac{\alpha}{\alpha-1} s - b\right] - c. \end{aligned} \tag{11}$$

Given free entry of firms, it is then easy to show the following remark.

Remark 2 *The labor market tightness in the high technology sector, θ_H, is increasing in the segmentation threshold s and in the productivity κ, while it is decreasing in the coefficient α of the Pareto distribution of skills, in the cost of opening a vacancy c, in the unemployment benefit b, in workers' bargaining power β, in the interest rate r and in the job separation rate δ.*

When firms decide whether to post a vacancy, they do not know with whom they will be matched, so they decide according to the expected workers' productivity, which, in the high technology sector, depends on the distribution of types. Hence, when there are more highly skilled workers, i.e. α decreases, or the output of each job κ increases, the expected productivity of matches increases too. As a result, more vacancies are posted, and the labor market tightness increases.

The analysis so far shows that keeping track of the labor market tightness in the two sub-markets and the segmentation cut-off is enough to fully describe an equilibrium. Hence,

Definition 1 *$(\theta_L^*, \theta_H^*, s^*)$ are an equilibrium if:*

a) *for $x < s^*$, $U_L(x) > U_H(x)$ and for $x \geq \max[1, s^*]$, $U_L(x) \leq U_H(x)$;*

b) *θ_L^* is such that $V(L) = 0$;*

c) *θ_H^* is such that $V(H) = 0$.*

The equilibrium can be of two types. When $\underline{y}$ is low relative to κ, $s^* \leq 1$ and only high technology vacancies are posted. For higher values of $\underline{y}$, s^* is interior and both technologies are adopted.

The properties of (10) and (11) imply that an equilibrium always exits and it is unique, as stated in the following Proposition:

Proposition 2 *Giving the parameters $c \in \mathbb{R}^{++}$, $0 \leq b < \underline{y}$, $\beta \in (0,1)$, $\kappa > 0$ and a matching function satisfying the Inada conditions, there always exists a unique equilibrium.*

The equilibrium can be described in the following way. First, $V(L) = 0$ gives the unique value of θ_L that is compatible with the equilibrium in that sector of the labor market. Second, Remark 2 implies that there is an increasing function $s = JC(\theta_H)$ that ensures $V(H) = 0$. Finally, given θ_L, (7) implies that there exists a strictly decreasing function $s = S(\theta_H)$ that describes the unique equilibrium level of s. This curve is strictly decreasing until 1, after which it is flat. Then, we are left with a system of two equations in two unknowns, which is represented graphically in Figure 1. The intersection between the two curves gives the equilibrium value of the labor market tightness θ_H^* and the segmentation threshold s^*.

This characterization of the equilibrium points out that the model's tractability stems from the fact that the distribution of skills of workers employed in the low technology sector does not affect the labor market tightness in that sector. This is due to two key features of the model: production there is independent on workers' types and the matching function displays constant returns to scale, so that in equilibrium market size does not affect matching opportunities.

Figure 1 about here.

Comparative Statics. The effect of the inequality of the distribution of skills, α, on the equilibrium is straightforward, since it enters only in (11):

Proposition 3 *In an equilibrium where both types of vacancies are posted, a decrease in the coefficient α of the Pareto distribution of skills decreases the segmentation threshold, s^*, and the unemployment rate in the high technology sector, u_H^*, while it increases the labor market tightness θ_H^*. Furthermore, the labor market tightness and the unemployment rate in the low technology sector, θ_L^* and u_L^*, do not change.*

Figure 2 gives a graphical representation of the Proposition. If there are more highly skilled workers, the expected productivity in the high technology sector increases. This induces more firms to post vacancies in this sector, increasing the labor market tightness. Hence, it becomes easier to find a high technology job. As a result, more workers look for a job in this sector, and s^* decreases. If α gets very low, s^* reaches 1, and thereafter only high technology vacancies are posted.

Figure 2 about here.

In the next Proposition, I analyze the effect on the equilibrium of an increase of κ, which can be interpreted as skill biased technological change:

Proposition 4 *In an equilibrium where both types of vacancies are posted, an increase in κ decreases the segmentation threshold s^*, while the labor market tightness in both sectors, θ_L^* and θ_H^*, and the unemployment rates, u_L^* and u_H^*, do not change.*

This result stems from the fact that the segmentation threshold s^* and the productivity parameter κ enter in both (7) and (11), and always as κs^*. Hence, their product must be constant in an equilibrium where both types of vacancies are posted, which means that s^* must decrease proportionally to the increase of κ.

Figure 3 depicts such changes. After an increase in κ, jobs in the high technology sector become more productive, and θ_H^* tends to increase. Since θ_L^* does not change, s^* goes down: workers of lower ability start to look for a high technology job, and θ_H^* tends to decrease. Overall, the effect on θ_H^* of technological change is offset by the effect induced on sorting via a reduction in s^*.

Figure 3 about here.

Hence, after skill biased technological change, the size of the high technology sector is bigger, and also the surplus generated there, so that wages increase. However, the labor market tightness and, by (9), the unemployment rate in the two sectors do not change. Of course, since fewer workers now are in the low technology sector where unemployment is higher, total unemployment is reduced.

The fact that the labor market tightness does not change in the sector that benefits from technological progress hinges on the fact that the expected productivity of the high technology sector is linear in κs^*. This is due to the particular skill distribution assumed here. Indeed, it would be enough to (realistically) assume a Pareto distribution with an upper bound of workers' level of skills for an increase in κ to increase the labor market tightness and to decrease unemployment in the high technology sector.

3 Constrained Efficiency

I will show in this section that the decentralized equilibrium is in general not efficient even when the surplus sharing rule used to determine wages satisfies the Hosios (1990) condition.

For simplicity, in this section, I assume that the meeting process in each sector of the labor market is governed by a Cobb-Douglas matching function $m(\theta_y) = \theta_y^{1-\gamma}$, with $\gamma \in (0,1)$. I denote the segmentation cut-off that the social planner sets to allocate workers to sub-markets as $\widetilde{s}$.

A social planner that maximizes steady state welfare solves the following maximization problem:[6]

$$\max_{\theta_L,\theta_H,\widetilde{s}} \int_1^{\infty} e^{-rt} \left\{ \underline{y} \left[\left(1 - \frac{1}{\widetilde{s}^{\alpha}}\right) - \mathbf{u}_L \right] + (b - c\theta_L)\mathbf{u}_L + \frac{\alpha\kappa\widetilde{s}}{\alpha-1} \left[\frac{1}{\widetilde{s}^{\alpha}} - \mathbf{u}_H \right] + (b - c\theta_H)\mathbf{u}_H \right\} dt, \tag{12}$$

subject to

$$\dot{\mathbf{u}}_L = \delta \left(1 - \frac{1}{\widetilde{s}^{\alpha}}\right) - \mathbf{u}_L(\delta + \theta_L^{1-\gamma}), \tag{13}$$

$$\dot{\mathbf{u}}_H = \delta \frac{1}{\widetilde{s}^{\alpha}} - \mathbf{u}_H(\delta + \theta_H^{1-\gamma}). \tag{14}$$

In words, the social planner maximizes the expected output in each sector net of the unemployment benefits and search costs incurred. Since in the low technology sector all agents are equally productive, the novelty with respect to a standard model with homogeneous agents is only the fact that the size of the sector is endogenous, via the segmentation threshold $\widetilde{s}$. In the high technology sector instead there is also an additional effect via average productivity, given by $\alpha\kappa\widetilde{s}/(\alpha-1)$, which depends on the composition of the workforce seeking a job there. Finally, the two constraints take care of the evolution of the number of jobs due to job creation and job destruction, which, in steady state, must be balanced.

Let λ_L and λ_H be the co-state variables for the laws of motion (13) and (14) respectively. I will denote by $\widetilde{s}^*$, $\widetilde{\theta}_L^*$ and $\widetilde{\theta}_H^*$ the socially optimal level of the segmentation threshold and the labor market tightness in the low and in the high technology sector respectively.

[6] I write the planner problem using the mass of unemployed workers instead of unemployment rate as I did above because it turns out that this formulation is easier to solve and present. Of course, the results do not change whichever the specification chosen.

The optimal paths of unemployment, labor market tightness and segmentation satisfy (13), (14) and the following Euler conditions:

$$[\theta_L] \qquad \mathbf{u}_L c e^{-rt}/r + \lambda_L \mathbf{u}_L (1-\gamma)\widetilde{\theta}_L^{*-\gamma} = 0, \tag{15}$$

$$[\theta_H] \qquad \mathbf{u}_H c e^{-rt}/r + \lambda_H \mathbf{u}_H (1-\gamma)\widetilde{\theta}_H^{*-\gamma} = 0, \tag{16}$$

$$[\widetilde{s}] \qquad \frac{\alpha}{\widetilde{s}^{*\alpha+1}}\left[-\frac{e^{-rt}}{r}\left(\underline{y} - \frac{\alpha\kappa\widetilde{s}^*}{\alpha-1}\right) - \lambda_L\delta + \lambda_H\delta\right] - \frac{e^{-rt}\alpha\kappa}{r(\alpha-1)}\left[\frac{1}{\widetilde{s}^{*\alpha}} - \mathbf{u}_H\right] = 0, \tag{17}$$

$$[\mathbf{u}_L] \qquad \left(\underline{y} - b + c\widetilde{\theta}_L^*\right) e^{-rt}/r + \lambda_L(\delta + \widetilde{\theta}_L^{*1-\gamma}) = \dot{\lambda}_L, \tag{18}$$

$$[\mathbf{u}_H] \qquad \left(\frac{\alpha\kappa\widetilde{s}^*}{\alpha-1} - b + c\widetilde{\theta}_H^*\right)\frac{e^{-rt}}{r} + \lambda_H(\delta + \widetilde{\theta}_H^{*1-\gamma}) = \dot{\lambda}_H. \tag{19}$$

To solve the planner problem, I substitute λ_L from (15) into (18) and λ_H from (16) into (19), substitute (13) and (14) into (17), evaluate the outcome in the steady state, and rearrange to obtain

$$\underline{y} - b = c\frac{r + \delta + \gamma\widetilde{\theta}_L^{*1-\gamma}}{(1-\gamma)\widetilde{\theta}_L^{*-\gamma}}, \tag{20}$$

$$\frac{\alpha\kappa\widetilde{s}^*}{\alpha-1} - b = c\frac{r + \delta + \gamma\widetilde{\theta}_H^{*1-\gamma}}{(1-\gamma)\widetilde{\theta}_H^{*-\gamma}}, \tag{21}$$

$$\frac{b\delta + r\underline{y} + \gamma\underline{y}\widetilde{\theta}_L^{*1-\gamma}}{r + \delta + \gamma\widetilde{\theta}_L^{*1-\gamma}} = \frac{b\delta + r\frac{\alpha\kappa\widetilde{s}^*}{\alpha-1} + \gamma\widetilde{\theta}_H^{*1-\gamma}\frac{\alpha\kappa\widetilde{s}^*}{\alpha-1}}{r + \delta + \gamma\widetilde{\theta}_H^{*1-\gamma}} - \frac{\kappa\widetilde{s}^*}{\alpha-1}\frac{\widetilde{\theta}_H^{*1-\gamma}}{\delta + \widetilde{\theta}_H^{*1-\gamma}}. \tag{22}$$

However, the solution of the decentralized equilibrium with the same Cobb-Douglas matching function is given by the following system of equations

$$\underline{y} - b = c\frac{r + \delta + \beta\theta_L^{*1-\gamma}}{(1-\beta)\theta_L^{*-\gamma}}, \tag{23}$$

$$\frac{\alpha\kappa s^*}{\alpha-1} - b = c\frac{r + \delta + \beta\theta_H^{*1-\gamma}}{(1-\beta)\theta_H^{*-\gamma}}, \tag{24}$$

$$\frac{b\delta + br + \beta\theta_L^{*1-\gamma}\underline{y}}{r + \delta + \beta\theta_L^{*1-\gamma}} = \frac{br + b\delta + \kappa s^*\beta\theta_H^{*1-\gamma}}{r + \delta + \beta\theta_H^{*1-\gamma}}. \tag{25}$$

Comparing the two sets of conditions shows that the condition $\gamma = \beta$ developed in Hosios (1990) is not sufficient to guarantee that the solution of the social planner coincides with the decentralized solution.

Proposition 5 *The decentralized equilibrium is not efficient even if $\gamma = \beta$.*

In particular, it is interesting to note that, while the condition $\gamma = \beta$ would be sufficient to eliminate the externalities present in vacancy creation in the two sectors, the segmentation cut-off would in general differ. Hence, vacancy creation is inefficient, but only due to the effect of sorting on the expected productivity in each sector.

The intuition is the following: while workers decide where to look for a job depending on the value of search when unemployed, the social planner would like them to look for the job whose expected surplus is higher. In particular, there are two sources of inefficiency that emerge comparing (22) and (25): *(i)* the marginal worker decides where to look for a job just looking at his own productivity, while the social planner looks at how the marginal worker changes the expected productivity in the high technology sector, and *(ii)*, the social planner takes into account the effect of expected productivity on the employment opportunities in the high technology sector, as reflected in the last term of the RHS of (22).

However, since the two effects are nested, a direct comparison of sorting in the decentralized equilibrium with the socially optimal threshold is difficult, and it does not yield a precise characterization of the sources of inefficiency.

4 Decentralizing the Optimal Allocation

To gather insights on the nature and sign of the different search externalities, in this section I derive the Pigouvian tax scheme that decentralizes the social optimum.

When the social planner sets a (possibly type and sector specific) tax on wages $\tau(x,y)$ for $y = L, H$, the value function of workers employed in sector $y = L, H$ is

$$rE(x,y) = \frac{b(r+\delta) + (r+\theta_y^{1-\gamma})[w(x,y) - \tau(x,y)]}{r+\delta+\beta\theta_y^{1-\gamma}},$$

while the value function for unemployed workers looking for a job in sector $y = L, H$ is

$$rU(x,y) = \frac{b(r+\delta) + \theta_y^{1-\gamma}[w(x,y) - \tau(x,y)]}{r+\delta+\beta\theta_y^{1-\gamma}}. \tag{26}$$

On the contrary, the value functions of filled jobs and of vacancies do not change but from the fact that the cost of opening the vacancy now is $c + \sigma(y)$ for $y = H, L$, where $\sigma(y)$ is a sector specific subsidy that the planner might set. Wages are pinned down by Nash bargaining, which leads to

$$w(x,y) = \beta Y(x,y) + (1-\beta)\frac{[b+\tau(x,y)](r+\delta) + \beta\theta_y^{1-\gamma}Y(x,y)}{r+\delta+\beta\theta_y^{1-\gamma}}. \tag{27}$$

Substituting this expression into (26) gives:

$$rU(x,y) = \frac{b(r+\delta) + \beta\theta_y^{1-\gamma}[Y(x,y)-\tau(x,y)]}{r+\delta+\beta\theta_y^{1-\gamma}}, \tag{28}$$

which imply that the threshold $\hat{s}$ at which the labor market segments is determined by[7]

$$\frac{b(r+\delta) + \beta\theta_L^{1-\gamma}[\underline{y}-\tau(\hat{s},L)]}{r+\delta+\beta\theta_L^{1-\gamma}} = \frac{b(r+\delta) + \beta\theta_H^{1-\gamma}[\kappa\hat{s}-\tau(\hat{s},H)]}{r+\delta+\beta\theta_H^{1-\gamma}}. \tag{29}$$

Combining these results with the conditions for firms (2) and (3) and the steady state unemployment levels (8) and (9), the two free entry conditions can be written as

$$\underline{y} - \int_1^{\hat{s}} \tau(x,L)g(x)dF(x) - b = [c+\sigma(L)]\frac{r+\delta+\beta\theta_L^{1-\gamma}}{(1-\beta)\theta_L^{-\gamma}}, \tag{30}$$

$$\frac{\alpha\kappa\hat{s}}{\alpha-1} - \int_{\hat{s}}^{+\infty} \tau(x,H)g(x)dF(x) - b = [c+\sigma(H)]\frac{r+\delta+\beta\theta_H^{1-\gamma}}{(1-\beta)\theta_H^{-\gamma}}. \tag{31}$$

Conditions (29), (30) and (31) determine the optimal segmentation threshold, denoted by $\hat{s}^*$, and the labor market tightness in the low technology sector, $\hat{\theta}_L^*$, and in the high technology sector, $\hat{\theta}_H^*$. Furthermore, let $\hat{\theta}^*$ be the optimal proportion of low technology vacancies.

Now, I can establish the following result.

Proposition 6 *A tax scheme* $\{\sigma(y), \tau(x,y)\}_{y=L,H}$ *implements the optimal allocation if:*

[7] I am assuming that the tax rates are set such that $U(x,L)$ and $U(x,H)$ have a unique intersection. Of course, this is what the planner would do in order to induce the optimal allocation. See Proposition 6 below.

1) the subsidies given to vacancies satisfy:

$$\sigma(L) = \frac{1}{(1-\gamma)(r+\delta+\beta\hat{\theta}_L^{*1-\gamma})}\left[c(r+\delta)(\gamma-\beta)+\gamma(1-\beta)\underline{y}\hat{\theta}_L^{*1-\gamma}-\beta(1-\gamma)\underline{y}\hat{\theta}_L^{*1-\gamma}\right], \tag{32}$$

$$\sigma(H) = \frac{1}{(1-\gamma)(r+\delta+\beta\hat{\theta}_H^{*1-\gamma})}\left[c(r+\delta)(\gamma-\beta)+\kappa\frac{\alpha\hat{s}^*}{\alpha-1}\hat{\theta}_H^{*1-\gamma}\gamma(1-\beta)-\kappa\frac{\alpha\hat{s}^*}{\alpha-1}\hat{\theta}_H^{*1-\gamma}\beta(1-\gamma)\right], \tag{33}$$

$$\hat{\theta}^*\sigma(L)+(1-\hat{\theta}^*)\sigma(H)=0; \tag{34}$$

2) the taxes on wages $\tau(x,L)$ and $\tau(x,L)$ are such that $U(x,L)$ and $U(x,H)$ intersect only once, at $\hat{s}^*$; furthermore, they satisfy:

$$\int_1^{\hat{s}^*}\tau(x,L)g(x)dF(x)=0, \tag{35}$$

$$\int_{\hat{s}^*}^{+\infty}\tau(x,H)g(x)dF(x)=0, \tag{36}$$

$$\tau(\hat{s}^*,L) = -\frac{(\gamma-\beta)[\underline{y}(r+\delta)-b\delta]}{\beta(r+\delta+\gamma\hat{\theta}_L^{*1-\gamma})}-\frac{r[\underline{y}(r+\delta+\beta\hat{\theta}_L^{*1-\gamma})-b(r+\delta+\gamma\hat{\theta}_L^{*1-\gamma})]}{\beta\hat{\theta}_L^{*1-\gamma}(r+\delta+\gamma\hat{\theta}_L^{*1-\gamma})}, \tag{37}$$

$$\tau(\hat{s}^*,H) = -\frac{(\gamma-\beta)\left[\frac{\alpha\kappa\hat{s}^*}{\alpha-1}(r+\delta)-b\delta\right]}{\beta(r+\delta+\gamma\hat{\theta}_H^{*1-\gamma})}-\frac{r\left[\frac{\alpha\kappa\hat{s}^*}{\alpha-1}(r+\delta+\beta\hat{\theta}_H^{*1-\gamma})-b(r+\delta+\gamma\hat{\theta}_H^{*1-\gamma})\right]}{\beta\hat{\theta}_H^{*1-\gamma}(r+\delta+\gamma\hat{\theta}_H^{*1-\gamma})}-\frac{\kappa\hat{s}^*}{\alpha-1}+\frac{\kappa\hat{s}^*(r+\delta+\beta\theta_H^{*1-\gamma})}{\beta(\alpha-1)(\delta+\hat{\theta}_H^{*1-\gamma})}. \tag{38}$$

These conditions result immediately comparing (29), (30) and (31) with (20), (21) and (22). Of course, there are infinitely many tax schemes that induce the social planner's solution.

Conditions (32) and (33) illustrate the usual frictions present in a random matching economy. The first term in the square parenthesis in both conditions represents the **entry externality**, which is positive if $\gamma > \beta$. The second term is due to the **thick-market externality** deriving from the fact that firms look at their private value of opening a vacancy and do not internalize the value of search for the unemployed, which is positive. The last term represents the **congestion externality** resulting from firms ignoring the effect of their search effort on the rest of the firms in the market, which is negative.

The third condition, (34), ensures that subsidies are budget balanced.

When the Hosios (1990) condition is satisfied, i.e. $\gamma = \beta$, the entry externality goes to zero and the thick market and the congestion externality cancel each other out, as in models with homogeneous firms and workers.

Turning our attention to labor taxes, conditions (35) and (36) state that the tax should be budget balanced within each sector of the economy, not to alter firms' incentives to open vacancies. Of course, it would be possible to devise non-neutral taxes and then compensate each sector via subsidies, but I chose this specification to facilitate the analysis when $\gamma = \beta$, i.e. when subsidies should be zero, since, if not, in general either the taxes would not implement the social optimum, or they would not be budget balanced. This implies that there should be redistribution within sectors, since redistribution across sectors would induce a non-optimal vacancy creation level. Hence, taxation has to be type *and* sector contingent.

A direct implication of this is that a social planner cannot induce the optimal allocation by imposing different bargaining powers depending on the worker's type, or by imposing sector specific subsidies.

Since the value of search of the marginal agents, i.e. those workers that are crucial to determine the segmentation threshold, is different from the social surplus, some labor taxes have to be introduced. Let me now study in detail the different components of such taxes.

The first term of conditions (37) and (38) represents the **participation entry externality**, which is positive if $\gamma < \beta$, i.e. it has the opposite sign of firms' entry externality. Indeed, this externality is not present when the Hosios (1990) condition is satisfied, as it is the case for firms.

The second term of both expressions is a **participation thick-market (or segmentation) externality**, that is due to frictions and wage bargaining: workers look for a job depending on the value of search and not depending on social surplus. In other words, looking at (22) and (25), this externality emerges because, in determining the segmentation threshold, the marginal type in the decentralized equilibrium looks at the unemployment benefit, while the social planner looks at his productivity in the two sectors. Hence, a social planner would like to subsidize search in both sectors.

The sum of the two effects, i.e. the overall **participation externality**, is negative, implying that agents should be subsidized in both sector to have a correct search behavior, and more so in the high technology sector, where the difference between unemployment benefits and one's productivity is larger.

The third term of (38) represents the **composition search externality** that is due to the fact that workers of type $\hat{s}^*$ decide whether to look for a job in the high technology sector looking at the wage they would have there—which depends on their productivity $\kappa\hat{s}^*$—while the social planner looks at the effect of these workers on expected productivity, that is $\kappa\alpha\hat{s}^*/(1-\alpha) > \kappa\hat{s}^*$. Hence, a social planner would like to subsidize the marginal workers $\hat{s}^*$.

The fourth term of (38) derives from the fact that the marginal worker that decides to look for a job in the high technology sector takes into account only his productivity, $\hat{s}^*$, and not the fact that he is reducing the expected productivity in that sector, reducing the possibility to find a match for all job seekers'. Hence, a social planner would like to tax such a worker in order for him to internalize this **composition congestion externality**.

The sum of the third and fourth term in (38) represents the overall **composition externality**, which is positive. This means that the marginal workers' types in the high technology sector should be taxed to make them internalize the negative externality they exert on other participants of that segment of the market by inducing a lower average productivity. This externality is different than the participation externality because it emerges from the fact that agents have different skill levels. As such, it is present only in the high technology sector, and, furthermore, it does not vanish when the Hosios (1990) condition is satisfied. However, this externality goes to zero when α goes to infinity, because in that case the distribution of types becomes degenerate.

Before studying when the composition dominates the participation externalities or *vice versa*, it is important to understand the difference between a standard search framework and the one studied here. To do so, I will now rewrite $\tau(x, L)$ and $\tau(x, H)$ assuming that the Hosios (1990) condition holds, i.e. $\gamma = \beta$—indeed, in that case $\sigma(L) = \sigma(H) = 0$:

$$\tau(\hat{s}^*, L) = -\frac{r(y-b)}{\beta\hat{\theta}_L^{*1-\beta}}, \tag{39}$$

$$\tau(\hat{s}^*, H) = -\frac{r}{\beta\hat{\theta}_H^{*1-\beta}}\left(\frac{\alpha\kappa\hat{s}^*}{\alpha-1} - b\right) - \frac{\kappa\hat{s}^*}{\alpha-1} + \frac{\kappa\hat{s}^*(r+\delta+\beta\theta^{1-\beta})}{\beta(\alpha-1)(\delta+\hat{\theta}_H^{*1-\beta})}. \tag{40}$$

Studying (39) and (40), the following corollary deserves some attention:

Corollary 1 *If $\gamma = \beta$ and $r \to 0$, then $\tau(\hat{s}^*, L) = 0$ and $\tau(\hat{s}^*, H) > 0$.*

In other words, when we abstract form the frictions induced by Nash bargaining, the decentralized allocation is not optimal even when the Hosios (1990) condition is satisfied. Indeed, when $r \to 0$, workers value of being unemployed is just the unemployment benefit, as in a static model. Hence, all the effects of wage determination on workers' sorting disappear. In other words, if wages were set competitively, the resulting equilibrium would have the same properties as when $r \to 0$. Then, the high technology sector would be too big, and a social planner would like to tax the worst types searching for a job there to induce them to look for a job in the low technology sector.

Indeed, when $\gamma = \beta$ and $r \to 0$, only the composition and the composition-congestion externalities are present. However, the second one is stronger, so that agents in the high technology sector need to be taxed. On the contrary, production is constant in the low technology sector, so that these externalities do not emerge there.

This result is the counterpart of the work of Albrecht et al. (2010): when wages are set competitively, or future periods do not matter, only the composition externality is present, and too many workers look for a job in the high technology sector. Indeed, the participation externalities disappear, since they result from agents not looking at the social surplus. Similar results are obtained in models with multiple segmentation outcomes when the equilibrium is not fully segregated, as shown in Blázquez and Jansen (2008) and Uren (2006), because composition externalities emerge only when workers of different skill levels are present in a given sector of the labor market.

While the previous result stresses that there can be excessive vacancy creation in the high technology sector, the next Corollary shows that this is not necessarily the case.

Corollary 2 *If $\gamma = \beta$, $r > 0$ and $.5 \leq \beta < 1$, there exists a value of the Pareto index $\bar{\alpha}$ such that for all $\alpha > \bar{\alpha}$, $\tau(\hat{s}^*, H) < \tau(\hat{s}^*, L) < 0$.*

In general, the direction of inefficiency depends on the distribution of inequality, i.e. on α, and the bargaining power of workers (or, the efficiency of the matching function). To derive this result, first note that $\tau(\hat{s}^*, H)$ is decreasing in α. Furthermore, when $\alpha \to 1$, the tax goes to $+\infty$. When instead $\alpha \to +\infty$, the last two terms of (40) disappear. Hence, we need to study how $-(Y(x,y) - b)/\hat{\theta}_y^{1-\beta}$ changes in y. Using, (23) and (24), we can rewrite this as $-c(r + \delta + \beta\hat{\theta}_y^{1-\beta})/[(1-\beta)\hat{\theta}_y^{1-2\beta}]$, which is decreasing in $\hat{\theta}$ if and only if $\beta(1-\beta) + (2\beta - 1)\hat{\theta}_y^{1-\beta}(r + \delta + \beta\hat{\theta}_y^{1-\beta}) > 0$. A sufficient condition for this inequality to hold is $\beta > .5$.[8] Hence, when the efficiency of the matching function—or, the bargaining power of workers—is sufficiently high, the subsidy to workers in the high technology sector is higher (or, the tax is more negative). Since $\tau(\cdot)$ is continuous in α, the result follows.

The intuition behind this result is the following. When α is small, there are many high skilled workers, which means that the composition externality is very strong. In that case, the expected productivity in the high technology sector is very high, which stimulates vacancy creation, increasing the labor market tightness. As a result, the labor market conditions in the high technology sector are so good that it is easy to find a job for all participants in that segment. This attracts too many workers' types with respect to what a social planner would do. Hence, a positive tax should be levied to the marginal workers in order to make them look for a job in the low technology sector where they would generate a higher surplus.

[8] Note that most empirical estimates of the elasticity of constant returns to scale matching functions would satisfy this condition, as reported in Petrongolo and Pissarides (2001).

When α is high, there are mostly low skilled workers, so that the composition externalities are weak. On the contrary, the participation externalities (and, in particular the participation thick-market externality) are strong. This means that agents do not have the right incentives to search, so that participation should be subsidized in both sectors. However, this externality is stronger in the high technology sector, where agents are more productive. As a result, the decentralized segmentation cut-off is too high, and, to decentralize the optimal allocation, there should be more incentives to search in the high technology sector than in the low technology one.

Two comments are in order. First, the mechanism behind this result resembles the one present in models with endogenous search intensity, as Shimer and Smith (2001), where more productive agents do not search hard enough. In this framework, this is translated in the fact that they do not look for a job in the high technology sector of the labor market. Second, in order for the tax to be budget-balanced, more productive agents than the marginal types have to be taxed negatively in the low technology sector and positively in the high technology sector, implying a regressive tax in the former and a progressive tax in the latter.

Concluding, the direction of the inefficiency crucially depends on the distribution of workers' skills because it influences the strength of the participation and composition externalities. Hence, in an economy where skilled workers are prevalent, we should expect the decentralized equilibrium to be biased in their favor, and the opposite when they are scarce. Furthermore, minimum wages would either be (almost always) non-binding, or induce firms not to adopt the low technology at all, which is not efficient in most cases.

The Pigouvian tax scheme shows that, once market segmentation is taken into account in a non-trivial manner, it is hard to reconcile equilibrium and efficiency. The source of inefficiency is not the particular wage determination protocol I assumed, i.e. Nash bargaining, but rather the endogenous participation decision that, due to random search, affects average productivity. Indeed, if firms were allowed to post type contingent wages, the decentralized equilibrium would be efficient: since workers of each type would look for a job in separate labor market segments, participation decisions would not affect the labor market conditions in other sub-markets. When instead firms cannot commit to wages conditional on type, participation and composition externalities emerge. Furthermore, when wages are set via Nash bargaining, both the direction and extent of the discrepancy between efficient and decentralized allocation depend on the distribution of types in the economy. This analysis shows that less skilled workers might pose negative externalities on more productive agents by inducing a too big high technology sector, and, hence, by decreasing average productivity there.

5 Conclusions

In this paper, I studied the social welfare properties of matching markets with two-sided heterogeneity, workers that are allowed to direct their search and firms that cannot condition the contracts they offer on the partner they meet. This framework allows analyzing the impact of the matching technology and the protocol assumed to pin down wages on the allocation of workers to firms, and on vacancy creation.

In particular, the model I proposed here delivers a unique equilibrium with an endogenous segmentation threshold that changes continuously. The decentralized equilibrium is in general inefficient even when the Hosios (1990) condition is satisfied. The inefficiency results not only because of the standard thick-market and congestion externalities, i.e. the fact that workers do not internalize the social benefit of their search activity due to wage bargaining, but also because of the composition and participation externalities that emerge in a framework where workers have different skill levels and decide the type of job to look for. Surprisingly, the direction of the inefficiency depends on the distribution of workers' skills.

The model developed here provides a parsimonious framework that is tractable but yet rich enough to assess the interactions between technological choice and sorting of workers to jobs. In a model with a general distribution of skills, technological change would have an impact on wage and unemployment inequality. In such an economy, an increase in the supply of skills would generate an increase in the demand of skills even when technological change is exogenous. Such a multiplier effect might explain why countries exposed to the same technological shock might have different adoption rates without relying on institutional differences or directed technological change. Lastly, technological change would have an impact on the sector composition of the economy. Since jobs with different productivity have different resilience to shocks, such models could help in understanding the relationship between sector composition and GDP volatility documented in Burren and Neusser (2013). These issues are left for future research.

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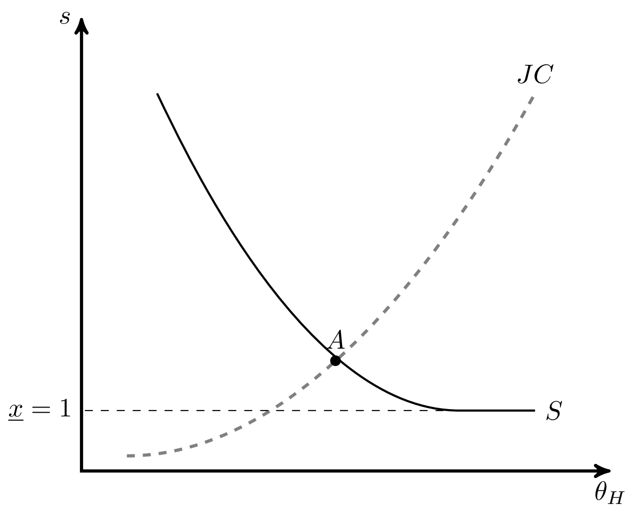

Figure 1: The two conditions for equilibrium.

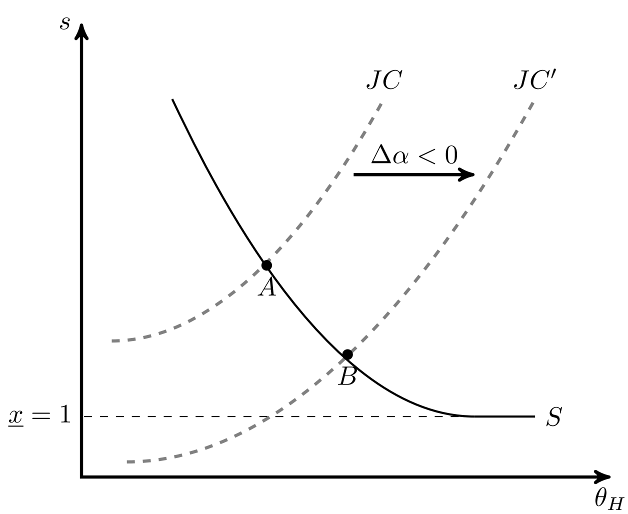

Figure 2: The equilibrium moves from A to B with a decrease in α., i.e. an increase in inequality, because the condition for job creation in the high technology sector shifts to the right.

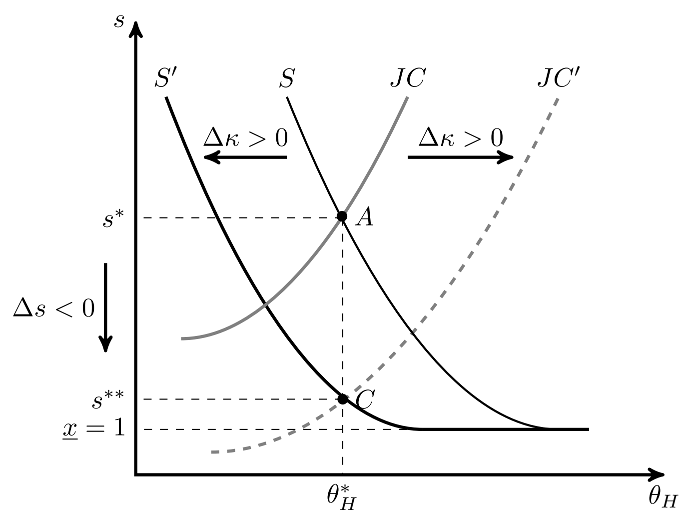

Figure 3: The equilibrium moves from A to C after an increase in κ, i.e. after skill biased technological change.